ELEMENTARY MATHEMATICS
FOR
WIRELESS OPERATORS

BY

W. E. CROOK

A M.I.E.E., A.F.R.Ae.S.

SECOND EDITION

LONDON

SIR ISAAC PITMAN & SONS, LTD.

1942

First Edition .	.	*June*, 1941
Reprinted .	.	*August*, 1941
,, .	.	*November*, 1941
Second Edition	.	*April*, 1942

SIR ISAAC PITMAN & SONS, LTD.
PITMAN HOUSE, PARKER STREET, KINGSWAY, LONDON, W.C.2
THE PITMAN PRESS, BATH
PITMAN HOUSE, LITTLE COLLINS STREET, MELBOURNE
UNITEERS BUILDING, RIVER VALLEY ROAD, SINGAPORE
27 BECKETTS BUILDINGS, PRESIDENT STREET, JOHANNESBURG

ASSOCIATED COMPANIES
PITMAN PUBLISHING CORPORATION
2 WEST 45TH STREET, NEW YORK
205 WEST MONROE STREET, CHICAGO
SIR ISAAC PITMAN & SONS (CANADA), LTD.
(INCORPORATING THE COMMERCIAL TEXT BOOK COMPANY)
PITMAN HOUSE, 381-383 CHURCH STREET, TORONTO

MADE IN GREAT BRITAIN AT THE PITMAN PRESS, BATH
D2—(T.206)

PREFACE

TO SECOND EDITION

THE author has always contended that few wireless operators will ever buy a book on mathematics. The reception accorded to this book, however, is not only most gratifying, but suggests that the foregoing opinion will have to be revised.

At the suggestion of teachers who have used the book, a section has been added on Elementary Arithmetic, and a number of examples with answers will be found after each chapter of the new Edition. It is hoped that these examples will help the student to put into practice what he has read in the text.

W. E. CROOK

March, 1942

PREFACE

TO FIRST EDITION

To all those being trained as wireless operators—

The question you will ask is: "Why should I bother about anything so repulsively dull as mathematics?"

Here is the answer.

You cannot qualify as a radio operator without knowing how your apparatus works. You cannot understand or even study wireless without some knowledge of simple mathematics. The mathematics you need to know are neither dull nor difficult. If you have this knowledge, your radio training will be enormously simplified and will be more pleasant and speedy.

If you have no knowledge of mathematics, you will be in much the same position as an engine fitter with no tools.

The author of this small work knows only too well the effect of lack of mathematics on radio operator trainees—it is a crushing handicap. The present volume is not a formal mathematical textbook. Indeed, it is quite likely to cause horror should it fall into the hands of the mathematical pundits. Casting modesty on one side, however, it will give the wireless operator all the mathematics he needs to know on his course, and will save both him and his instructors from those moments of despair so familiar in radio schools. The practical man has no time to make the study of mathematics an end in itself, and few have the necessary gifts for such a study. But if you cannot have the whole tool-box, it is better to have a few simple tools than none at all, and much can be done with them.

W. E. CROOK

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CHAPTER I

ARITHMETIC

We will assume that you know how to carry out addition, subtraction, multiplication and simple division. Long division and the multiplication of large numbers are more easily and quickly done by the use of logarithms, which will be fully explained in the next chapter.

Fractions. A fraction is the arithmetical way of stating the ratio between two quantities. Any measurable quantity can be divided into any number of parts. Take, for example, a distance of one mile. The mile can be divided into 8 equal parts called furlongs, so that a furlong is one-eighth ($\frac{1}{8}$) of a mile. Similarly, a mile is also equal to 1760 yards, 5280 feet or 63,360 inches.

Therefore

$$1 \text{ yard} = \frac{1}{1760} \text{ mile}$$

$$1 \text{ foot} = \frac{1}{5280} \text{ mile}$$

$$1 \text{ inch} = \frac{1}{63360} \text{ mile}$$

The lower figure of a fraction is called the denominator and the upper figure is called the numerator. The denominator shows what part of the whole is being considered, and the numerator indicates how many of such parts the fraction denotes. Thus $\frac{2}{5}$ means that fifth parts are being considered, and that two such parts are taken.

When the numerator is less than the denominator, the expression is called a proper fraction and its value is clearly less than 1.

When the numerator is more than the denominator the fraction is greater than 1, and is equivalent to a whole number plus a proper fraction. Thus, $\frac{7}{5}$ means that seven fifth-parts are taken. Five fifths are equal to 1, leaving two fifths, so that $\frac{7}{5}$ represents one whole unit plus $\frac{2}{5}$ of a unit, and may be written $1\frac{2}{5}$.

Similarly $$\frac{14}{9} = 1\frac{5}{9}$$

$$\frac{14}{3} = 4\frac{2}{3}$$

$$\frac{100}{7} = 14\frac{2}{7}$$

Addition and Subtraction of Fractions. If you go into a shop and buy half a pound of cheese, and then go into another shop and buy a quarter of a pound of cheese, you will have acquired three-quarters of a pound of cheese. Arithmetically—

$$\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

Analysing this result, it will be seen that in order to add fractions they must first be modified to have the same denominators. What we have really done is to say that $\frac{1}{2}$ lb. is equal to two quarters, and the other quarter makes three-quarters, i.e.

$$\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$$

Now suppose we want to add together $\frac{1}{4}$ and $\frac{1}{5}$.

$$\frac{1}{4} \text{ is the same as } \frac{5}{20}$$

$$\frac{1}{5} \text{ is the same as } \frac{4}{20}$$

Therefore $$\frac{1}{4} + \frac{1}{5} = \frac{5}{20} + \frac{4}{20} = \frac{9}{20}$$

The 20 is in this case obtained simply by multiplying the two denominators together. Take another example—

$$\frac{4}{9} + \frac{3}{8}$$

The common denominator will be $8 \times 9 = 72$. $\frac{4}{9}$ is $\frac{32}{72}$ (obtained by multiplying both numerator and denominator by 8, which of course does not alter the *value* of the fraction). Similarly, $\frac{3}{8}$ is $\frac{27}{72}$.

Therefore $$\frac{4}{9} + \frac{3}{8} = \frac{32}{72} + \frac{27}{72} = \frac{59}{72}$$

In words, this result means that four ninth-parts of something plus three eighth-parts is equivalent to fifty-nine seventy-second parts.

The common denominator may be less than the figure obtained by multiplication of the separate denominators. Take, for instance—

$$\frac{5}{12} + \frac{3}{20}$$

A little reflection will show that the smallest number which can be divided exactly by both 12 and 20 is 60.

Therefore $$\frac{5}{12} + \frac{3}{20} = \frac{25}{60} + \frac{9}{60} = \frac{34}{60} \text{ or } \frac{17}{30}$$

The 60 in this case is called the Least Common Multiple (L.C.M.) of 12 and 20. It is not absolutely necessary to find the L.C.M. before adding fractions. In the last example, we could have written—

$$\frac{5}{12} + \frac{3}{20} = \frac{100}{240} + \frac{36}{240} = \frac{136}{240}$$

The result $\frac{136}{240}$ can clearly be reduced because 136 and 240 can both be divided by 2, 4 or 8. Dividing both by 8, we get $\frac{17}{30}$ which is the same answer.

The subtraction of fractions consists of the same process—reduction to a common denominator and subsequent subtraction of one numerator from the other. Thus—

$$\frac{5}{12} - \frac{3}{20} = \frac{25}{60} - \frac{9}{60} = \frac{16}{60}$$

$\frac{16}{60}$ can be further reduced by dividing 4 into both figures $= \frac{4}{15}$.

In dealing with two fractions, the quickest way is shown in the following illustration. The figures are denoted by letters, which may have any value—

$$\frac{a}{b} + \frac{c}{d} = \frac{(a \times d) + (b \times c)}{b \times d}$$

Applying this rule to an earlier example—

$$\frac{5}{12} + \frac{3}{20} = \frac{(5 \times 20) + (3 \times 12)}{(12 \times 20)}$$

$$= \frac{100 + 36}{240}$$

$$= \frac{136}{240}$$

$$= \frac{17}{30}$$

Similarly
$$\frac{5}{13}+\frac{7}{10}=\frac{50+91}{130}$$
$$=\frac{141}{130}$$
$$=1\frac{11}{130}$$

When adding up more than two fractions it is better to take the L.C.M. of the denominators first, unless the figures are very simple.

As already mentioned, the L.C.M. (Least Common Multiple) of a set of numbers is the smallest number which is divisible by all of them without remainder. As every number is a multiple of two or more of the figures 1, 2, 3, 5, and 7, the method of finding the L.C.M. is fairly self-evident. For example, consider 12 and 20.

$$12 = 2 \times 2 \times 3$$
$$20 = 2 \times 2 \times 5$$

The L.C.M. must clearly have 2×2 as one of its factors, and it must also contain 3 and 5 as factors. Therefore—

$$\text{L.C.M.} = 2 \times 2 \times 3 \times 5 = 60$$

Taking another case, consider the L.C.M. of 18, 30, and 45.

$$18 = 2 \times 3 \times 3$$
$$30 = 2 \times 3 \times 5$$
$$45 = 3 \times 3 \times 5$$

The highest power of 2 which occurs is 2—in 18 and 30.
The highest power of 3 which occurs is 3^2—in 18 and 45.
The highest power of 5 which occurs is 5—in 30 and 45.
There are no other factors.
Therefore L.C.M. $= 2 \times 3 \times 3 \times 5 = 90$.

EXAMPLE.
$$\frac{13}{40}-\frac{33}{112}+\frac{51}{140}+\frac{18}{35}$$

$$40 = 2 \times 2 \times 2 \times 5$$
$$112 = 2 \times 2 \times 2 \times 2 \times 7$$
$$140 = 2 \times 2 \times 5 \times 7$$
$$35 = 5 \times 7$$

$$\text{L.C.M.} = 2 \times 2 \times 2 \times 2 \times 5 \times 7 = 560$$

To convert all fractions to the same denominator, divide the denominator into the L.C.M. and multiply the numerator by the answer. Thus, taking $\frac{13}{40}$, we have to multiply both numerator and denominator by a number which will produce 560 as the new denominator. This is clearly $\frac{560}{40} = 14$.

The fraction then becomes $\frac{13}{40} \times \frac{14}{14} = \frac{182}{560}$. Continuing thus—

$$\frac{13}{40} - \frac{33}{112} + \frac{51}{140} + \frac{18}{35} = \frac{182 - 165 + 204 + 288}{560}$$
$$= \frac{674 - 165}{560}$$
$$= \frac{509}{560}$$

Multiplication and Division of Fractions. If one fraction is multiplied by another, it means that we are taking a fraction of a fraction. For example, it is clear that half a half is one-quarter, i.e.

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

The result is obtained by multiplying the numerators together to get the new numerator, and multiplying the denominators together to get the new denominator. Thus—

$$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$$

$$\frac{2}{5} \times \frac{3}{7} = \frac{6}{35}$$

$$\frac{9}{13} \times \frac{1}{2} = \frac{9}{26}$$

When multiplying fractions it is often convenient to make cancellations, i.e. dividing the numerator and denominator by a common factor, or striking out factors which occur in both.

For example $$\frac{20}{31} \times \frac{4}{5}$$

The 5 can be divided into the 20, giving

$$\frac{\overset{4}{\cancel{20}}}{31} \times \frac{4}{\cancel{5}} = \frac{16}{31}$$

The process is, of course, equivalent to dividing both numerator and denominator by 5.

Taking another example, the working would be as follows—

$$\frac{\cancel{7}}{16} \times \frac{\cancel{4}}{\underset{\cancel{3}}{\cancel{21}}} \times \frac{\cancel{3}}{\cancel{4}} = \frac{1}{16}$$

The steps are: Cancel the two 4's.
Cancel the 7 into the 21 making 3.
Cancel the two 3's.

The process of dividing one fraction by another consists of inverting the divisor fraction and multiplying. The reason for this can be seen if a simple case is considered. The whole of something contains 8 equal eighth parts. A half therefore contains 4 equal eighth parts. Thus if we divide a half by an eighth the answer will be 4.

$$\frac{1}{2} \div \frac{1}{8} = \frac{1}{2} \times \frac{8}{1} = \frac{8}{2} = 4$$

Further examples are as follows—

$$\frac{15}{16} \div \frac{3}{8} = \frac{\overset{5}{\cancel{15}}}{\underset{2}{\cancel{16}}} \times \frac{\cancel{8}}{\cancel{3}} = \frac{5}{2} = 2\frac{1}{2}$$

$$\frac{3}{8} \div \frac{15}{16} = \frac{\cancel{3}}{\cancel{8}} \times \frac{\overset{2}{\cancel{16}}}{\underset{5}{\cancel{15}}} = \frac{2}{5}$$

$$\frac{3}{4} \times \frac{5}{8} \div \frac{5}{24} = \frac{3}{4} \times \frac{\cancel{5}}{\cancel{8}} \times \frac{\overset{3}{\cancel{24}}}{\cancel{5}} = \frac{9}{4} = 2\frac{1}{4}$$

Decimals. The decimal notation is an alternative method of expressing fractional quantities. The whole number (if any) is separated from the fractional part by a dot. The first figure following the dot indicates the number of tenth parts. The second figure after the dot indicates the number of hundredth parts, the next figure the number of thousandth parts, and so on. Thus one-tenth part of something is written as 0·1 (read "Point one").

A hundredth part is written as 0·01 (read "Point nought one"), and a thousandth part as 0·001.

A ten-thousandth part is 0·0001, and in most engineering calculations it is rarely necessary to go beyond four figures following the point.

Any fraction can be converted into a decimal by merely dividing the numerator by the denominator, which is, of course, exactly what a fraction means. A whole number can be regarded as having an indefinite number of 0's following the decimal point, for example, the number 3 can be written as 3·00000000..... for division purposes. Let us convert a few simple fractions into decimals.

$$\frac{1}{4} = \frac{4)1{\cdot}0000}{{\cdot}25} = 0{\cdot}25$$

$$\frac{1}{2} = \frac{2)1{\cdot}000}{{\cdot}5} = 0{\cdot}5$$

$$\frac{3}{8} = \frac{8)3{\cdot}0000}{{\cdot}375} = 0{\cdot}375$$

$$\frac{13}{16} = \frac{4)13{\cdot}0000}{\begin{array}{l}4)3{\cdot}25\\ \hline {\cdot}8125\end{array}} = 0{\cdot}8125$$

$$\frac{1}{12} = \frac{12)1{\cdot}00000\ldots.}{{\cdot}083333\ldots.} = 0{\cdot}0833 \text{ (to 4 places)}$$

This is called a recurring decimal, and as the 3's go on for ever, it cannot be expressed exactly, but, as previously mentioned, the first four figures after the point give quite sufficient accuracy for most purposes.

$$\frac{22}{7} = \frac{7)22{\cdot}0000000\ldots..}{3{\cdot}1428571\ldots.} = 3{\cdot}1428 \text{ (to 4 places).}$$

This is also a type of recurring decimal, the figures 142857 repeating themselves indefinitely.

The addition and subtraction of decimals is carried out exactly as with whole numbers, but the decimal points must, of course, be kept under each other.

Examples. Add together 3·57, 14·258, and 106·031.

$$\begin{array}{r} 3{\cdot}570 \\ 14{\cdot}258 \\ 106{\cdot}031 \\ \hline 123{\cdot}859. \\ \hline\hline \end{array} \quad \textit{Answer.}$$

Add together 8·963 and 21·72.

$$\begin{array}{r} 8{\cdot}963 \\ 21{\cdot}720 \\ \hline 30{\cdot}683. \\ \hline\hline \end{array} \quad \textit{Answer.}$$

Subtract 3·471 from 7·139.

$$\begin{array}{r} 7{\cdot}139 \\ 3{\cdot}471 \\ \hline 3{\cdot}668. \\ \hline\hline \end{array} \quad \textit{Answer.}$$

The multiplication and division of decimals is best carried out by the use of logarithms, as will be shown in the next chapter.

Note, however, that the decimal notation provides an instantaneous means of dividing or multiplying any number by any multiple of 10. The process merely involves a movement of the

decimal point, to the left for division, and to the right for multiplication. Take, for example, the number 396.

$$396 = 396{\cdot}0$$

$$\frac{396}{10} = 39{\cdot}6$$

$$396 \times 100 = 39600{\cdot}0$$

$$\frac{396}{1000} = 0{\cdot}396$$

On Calculations. Scientific and engineering work is about 1 per cent intuition and 99 per cent calculation. Anyone who studies scientific or engineering subjects must therefore be able to calculate with reasonable facility. Playing a piano whilst wearing boxing gloves is almost as hopeless a task as studying these subjects without being able to calculate. As an actual or potential wireless operator, you are certain sooner or later to have to work some problem out for yourself. Could anything be more humiliating than to find that although you have the facts, figures and formulae, you are either unable to make the calculation or your calculating processes are so decrepit that you cannot place any reliance upon the answer? Arithmetic is not therefore the tiresome invention of an Education Committee, but a means to an end. In this chapter you have been reminded of some of the simple arithmetic you learnt at school but may have forgotten. Now let us go on and see how calculating is really done—it is not difficult.

EXERCISES I

Evaluate the following expressions—

1. $\frac{1}{4} + \frac{1}{2} - \frac{3}{16}$.
2. $\frac{2}{9} + \frac{11}{24} + \frac{5}{12} - \frac{1}{8} - \frac{5}{16}$.
3. $\frac{7}{32} + \frac{7}{10} + \frac{3}{8}$.
4. $2\frac{4}{9} + 6\frac{5}{12} - 3\frac{1}{2}$.
5. $\frac{13}{20} \times \frac{5}{78} \div \frac{2}{7}$.
6. $\frac{11}{12} \times \frac{2}{3}$.
7. $3\frac{3}{8} \times 1\frac{1}{3} \div 2\frac{3}{16}$.
8. Add together 0·0861, 3·2, 5·714, 6·0032.
9. Subtract 3·9186 from 4·0010.
10. Convert the following fractions into decimals—

$$\frac{7}{8} \qquad \frac{13}{20} \qquad \frac{100}{61}$$

CHAPTER II

ARITHMETIC (*contd.*)

It is sometimes necessary to use brackets in making calculations, and when removing the brackets one has to be careful about + and — signs. The rule is very simple.

A + sign in front of a bracket leaves all signs within the bracket unchanged.

A — sign in front of a bracket changes all signs within the bracket.

Here is an example—

(*a*)
$$\begin{aligned} & 100 + (40 - 10 + 2) \\ = \; & 100 + 40 - 10 + 2 = 132 \end{aligned}$$

(*b*)
$$\begin{aligned} & 100 - (40 - 10 + 2) \\ = \; & 100 - 40 + 10 - 2 = 68 \end{aligned}$$

Curly and square brackets are also used if it is necessary to employ more than one set. The same rules apply in every case, as the following examples will show—

(*a*)
$$\begin{aligned} & 150 + \{70 - (36 + 4 - 7)\} \\ = \; & 150 + \{70 - 36 - 4 + 7\} \\ = \; & 150 + 37 = 187 \end{aligned}$$

(*b*)
$$\begin{aligned} & 48 + [100 - \{18 - (36 + 4)\}] \\ = \; & 48 + [100 - \{18 - 40\}] \\ = \; & 48 + [100 - \{-22\}] \\ = \; & 48 + [100 + 22] \\ = \; & 48 + 122 = 170 \end{aligned}$$

Multiplication or division may be indicated by means of brackets, the multiplier or fraction being placed in front of the bracket—

(*a*)
$$\begin{aligned} & 90 + 2\{70 - 2(40 - 5)\} \\ = \; & 90 + 2\{70 - 80 + 10\} \\ = \; & 90 + 2\{0\} = 90 \end{aligned}$$

(*b*)
$$\begin{aligned} & 55 - \tfrac{1}{3}\{75 + 3(11 - 30)\} \\ = \; & 55 - \tfrac{1}{3}\{75 + 3(-19)\} \\ = \; & 55 - \tfrac{1}{3}\{75 - 57\} \\ = \; & 55 - \tfrac{1}{3}\{18\} \\ = \; & 55 - 6 = 49 \end{aligned}$$

Powers and Roots. The "square" of a number is the result of multiplying it by itself. Instructions to do this are given by a small "2" placed above and to the right of the number. The "2" is called an index. Thus—

$$3^2 = 3 \times 3 = 9$$

$$10^2 = 10 \times 10 = 100$$

$$(2{\cdot}5)^2 = 2{\cdot}5 \times 2{\cdot}5 = 6{\cdot}25$$

$$\left(\frac{1}{8}\right)^2 = \frac{1}{8} \times \frac{1}{8} = \frac{1}{64}$$

The index "3" means that the number is to be multiplied by itself twice, and the result is called the "cube" of the number.

$$3^3 = 3 \times 3 \times 3 = 27$$

$$10^3 = 10 \times 10 \times 10 = 1000$$

$$(2{\cdot}5)^3 = 2{\cdot}5 \times 2{\cdot}5 \times 2{\cdot}5 = 15{\cdot}625$$

$$\left(\frac{1}{8}\right)^3 = \frac{1}{8} \times \frac{1}{8} \times \frac{1}{8} = \frac{1}{512}$$

Indices greater than 3 have no special name but are simply called the 4th power, 5th power, etc., of the number. Thus, 10^6 is read as "Ten to the sixth power," or just "Ten to the sixth."

The extraction of roots is the reverse process. The root of a number may be shown either by the root sign $\sqrt{\ }$ or by means of a fractional index. Thus—

$$\sqrt{9} = 9^{\frac{1}{2}} = 3$$

$$\sqrt[3]{125} = 125^{\frac{1}{3}} = 5$$

$$\sqrt[4]{16} = 16^{\frac{1}{4}} = 2$$

A negative index, i.e. one with a minus sign in front of it, may seem a little odd at first, but it is very easy to deal with. Change its sign to $+$ and put the number under 1. Thus—

$$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$$

$$27^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{27}} = \frac{1}{3}$$

$$10^{-1} = \frac{1}{10^1} = \frac{1}{10}$$

Now in all these cases the numbers have been simple and the results easy to get. In engineering and radio work it need hardly

be said that our calculations do not tumble out in whole numbers so conveniently. For instance, we may have to work out a thing like this—

$$\frac{240}{36} \times 2{\cdot}7182^{-0{\cdot}048}$$

or
$$\left(\frac{6{\cdot}28 \times 3{\cdot}579 \times 10^6 \times 47}{10^6}\right)^2$$

The second expression is easy enough, though rather laborious, but the first one apparently presents a knotty problem. How are we to deal with $2{\cdot}7182^{-0{\cdot}048}$?

i.e.
$$\frac{1}{2{\cdot}7182^{0{\cdot}048}}$$

We know that $2{\cdot}7182^2$ would be $2{\cdot}7182 \times 2{\cdot}7182$, but an index of 0·048 is something quite different. The key to the riddle is the use of logarithms.

Logarithms have one purpose only—to facilitate and speed up arithmetical calculations, and this they do in no uncertain way. If you have the idea, as so many have, that logarithms constitute a form of mathematical athleticism, and are specially designed for the confusion of radio students—forget it. You can learn to use logs. very quickly, and if you have to do any calculations worthy of the name, the log. table can be your best friend. There is no need to bother about how logs. are calculated or the theory of logs. No one insists that before learning to play a piano you must yourself construct the instrument, and in using logs. the same argument applies. You will now be told all you need to know about logs. and shown how to use them. Before reading further, you must have a table of logs. in front of you for reference—any engineer's pocket-book contains this.

Logarithms. This is the definition of a logarithm. The logarithm of a number to a given base is the power to which the base must be raised to produce that number.

The statement looks formidable at first glance, but actually the idea is very simple. All log. tables of "common" logs. are calculated to base 10. (There are also tables of "hyperbolic" logs. but we need not deal with these.) A few illustrations will show you at once what the definition means—

$$10^1 = 10$$

Therefore the logarithm of 10 is 1. This is written log. 10 = 1.

$$10^2 = 100$$

Therefore log. 100 = 2.

$$10^3 = 1000$$

Therefore log. 1000 = 3; and so on.

To get any number between, say, 10 and 100, it is clear that we have to raise 10 to some power between 1 and 2. Similarly, for any number between 100 and 1000 the log. will be between 2 and 3. For a number between 1 and 10, the log. will be between 0 and 1. ($10^0 = 1$, so that log. $1 = 0$.) And for a number less than 1, i.e. a decimal fraction, the log. will be negative. For example—

$$10^{-1} = \frac{1}{10} = 0{\cdot}1$$

Therefore log. $0{\cdot}1 = -1$.

$$10^{-2} = \frac{1}{10^2} = 0{\cdot}01$$

Therefore log. $0{\cdot}01 = -2$.

In these cases, the minus sign is written *over* the logarithmic whole number, like this—

$$\text{Log. } 0{\cdot}1 = \bar{1}$$
$$\text{Log. } 0{\cdot}01 = \bar{2}$$

These logs. are read as "Bar one" and "Bar two" and so on. The reason for this will be seen very shortly.

If you have not studied logs. before you will probably now be wondering what it is all about, the whole business no doubt seeming rather futile. But be patient and you will soon see how beautifully logs. can smooth out the most tiresome arithmetic for you. For the moment, let us make quite sure how to read off the log. of any number from the tables, and then we will see how to use them.

How to Read the Log. Tables. The left-hand column contains all the whole numbers from 10 to 99 and this, in conjunction with the other columns, is enough to obtain the log. of any number.

The next nine columns give you the logs. of numbers containing three significant figures, and the next five "difference" columns enable you to deal with a fourth significant figure. This is enough for almost all practical purposes. An example using each column in turn will make it clear. Let us take 21. The log. of a whole number is taken from the column headed 0. Opposite 21 you will see the figures 3222. Now we know that as 21 is between 10 and 100, the log. will be between 1 and 2. There is therefore never any need to consult the log. tables for the whole number part of the log. and actually of course it is not shown. The figures in the log. tables constitute the other part of the log. which is a decimal fraction.

Thus log. $21 = 1{\cdot}3222$. This means that $10^{1{\cdot}3222} = 21$. If we wanted the log. of 210, this would be $2{\cdot}3222$, and log. $2100 = 3{\cdot}3222$, and so on.

Now consider 21·1. In this case we take the log. from the column headed 1, and you will see that log. 21·1 = 1·3243.

Similarly, Log. 21·2 = 1·3263

Log. 21·3 = 1·3284

Log. 21·4 = 1·3304

Log. 21·5 = 1·3324

and Log. 21·9 = 1·3404

The five "difference" columns are not logs. but are the numbers to be added to the logs. for a fourth significant figure.

Suppose we want log. 21·11.

Log. 21·1 = 1·3243

In the difference column headed 1 we find the figure 2 opposite the 21 line. This means that if 1 is the fourth significant figure, 2 must be added to the log.

So that
$$
\begin{aligned}
\text{Log. } 21{\cdot}11 = {} & 1{\cdot}3243 \\
& \phantom{1{\cdot}324}2 \\
\hline
& 1{\cdot}3245
\end{aligned}
$$

Note that the "differences" are added straight on to the decimal part of the log.

Similarly, Log. 21·13 = 1·3249

If we want log. 21·12 this is obtained by what the mathematician likes to call "interpolation." This terrifying expression merely means that we take log. 21·12 as being half-way between log. 21·11 and log. 21·13.

Log. 21·11 = 1·3245

Log. 21·13 = 1·3249

So that Log. 21·12 = 1·3247

That is interpolation—quite simple. Sometimes you cannot get an exact half-way figure unless you make a five-figure log., but this is hardly ever necessary. For instance, take log. 15·52. You will see that

Log. 15·51 = 1·1906

and Log. 15·53 = 1·1911

So that log. 15·52 will be just about 1·19085. In practice, however, we can take it as 1·1908 or 1·1909, which is quite near enough and avoids the inconvenience of a fifth decimal in the log.

The logs. of numbers less than 10 are taken in the same way, but they will be decimal fractions only, because the log. of any

number between 1 and 10 is less than 1. Log. 2, for instance, is taken from the 0 column opposite 20.

$$\text{Log. } 2 = 0{\cdot}3010$$
$$\text{Log. } 3 = 0{\cdot}4771$$
$$\text{Log. } 4 = 0{\cdot}6021$$

The logs. of numbers less than 1, i.e. of decimal fractions, will be negative, but the log. is written as partly negative and partly positive. This may seem odd, but it is much more convenient when using logs., as you will see. Suppose we want log. 0·3. From the tables we see that log. 3 = 0·4771. Now, since 0·3 is less than 1, but more than 0·1, the log. will be between 0 and − 1. All we have to do is to write − 1, i.e. $\bar{1}$ in front of the log. of 3.

So that $$\text{Log. } 0{\cdot}3 = \bar{1}{\cdot}4771$$

The $\bar{1}$ is negative, but the 0·4771 is positive The actual log. is therefore

$$-1 + 0{\cdot}4771 = -0{\cdot}5229$$

i.e. $$10^{-{\cdot}5229} = 0{\cdot}3$$

If we wanted the log. of 0·03 this is found by changing the whole number part of the log. 0·03 is less than 0·1, but more than 0·01, so that the log. will be between − 1 and − 2.

$$\text{Log. } 0{\cdot}03 = \bar{2}{\cdot}4771$$

The actual log. is therefore

$$-2 + 0{\cdot}4771 = -1{\cdot}5229$$

i.e. $$10^{-1{\cdot}5229} = 0{\cdot}03$$

Similarly, $$\text{Log. } 0{\cdot}003 = \bar{3}{\cdot}4771$$
$$\text{Log. } 0{\cdot}0003 = \bar{4}{\cdot}4771$$

and so on.

A useful rule to remember is that the negative part of the log. is numerically one more than the number of 0's following the decimal point in the number whose log. is required. Thus, in the case of log. 0·0003 there are *three* 0's after the decimal point and the negative part of the log. is $\bar{4}$.

Now let us take a series of examples covering all possibilities.

EXAMPLE 1. Find logs. of 11·2, 1·12, and 112.

$$\text{Log. } 11{\cdot}2 = 1{\cdot}0492$$
$$\text{Log. } 1{\cdot}12 = 0{\cdot}0492$$
$$\text{Log. } 112 = 2{\cdot}0492$$

EXAMPLE 2. Find log. 7598.

The log. of 75·9 is 1·8802. There is no difference column for 8, and the differences for 7 and 9 are 4 and 5 respectively, so we can add 4 as being near enough.

$$\text{Log. } 75{\cdot}98 = 1{\cdot}8806$$

Therefore $$\text{Log. } 7598 = 3{\cdot}8806$$

since 7598 is more than 1000 but less than 10,000.

EXAMPLE 3. Find log. (6×10^6), i.e. the log. of 6 million.

From the tables $\text{Log. } 6 = 0{\cdot}7782$

Therefore $\text{Log. } (6 \times 10^6) = 6{\cdot}7782$

EXAMPLE 4. Find log. 364,597.

You will have seen that the columns of the log. table do not provide for more than four figures in the number whose log. is required. Actually this does not matter, because even when there are more than four figures we can get quite near enough to the true log. for all practical purposes. Treatment is as follows—

364,597 is almost 364,600.

From the tables the decimal part of the log. for the figures 3646 is 0·5618. As our number is more than 10^5 but less than 10^6 the whole number part of the log. will be 5. So that

$$\text{Log. } 364{,}597 = 5{\cdot}5618$$

EXAMPLE 5. Find log. 2,596,412.

Taking 2596 as the significant figures, the decimal part of the log. will be 0·4143. The number is more than 10^6 but less than 10^7, so that

$$\text{Log. } 2{,}596{,}412 = 6{\cdot}4143$$

At this point you may feel inclined to raise the objection that logs. are not accurate—it may seem rather drastic in the last example to ignore the last three figures of the number.

The explanation is that 412 is negligible in comparison with 2,596,412 and further, even if the 412 was taken into account, it *would not affect* the first four figures of the decimal part of the log. If we used 7-figure log. tables a slightly more accurate log. could be obtained, but for the majority of engineering calculations and for all those you are likely to do, 4-figure log. tables are adequate. More will be said about this point shortly.

EXAMPLE 6. Find log. 0·02173.

For the figures 2173 the decimal part of the log. is 0·3371.

$$\text{Log. } 0{\cdot}02173 = \bar{2}{\cdot}3371$$

EXAMPLE 7. Find log. 1·011.

For the figures 1011 the decimal part of the log. is 0·0047.

$$\text{Log. } 1{\cdot}011 = 0{\cdot}0047$$

Note again that moving the decimal point in the number merely involves a change in the whole number part of the log.

Thus
$$\begin{aligned} \text{Log. } 10{\cdot}11 &= 1{\cdot}0047 \\ \text{Log. } 101{\cdot}1 &= 2{\cdot}0047 \\ \text{Log. } 0{\cdot}1011 &= \bar{1}{\cdot}0047 \\ \text{Log. } 0{\cdot}01011 &= \bar{2}{\cdot}0047 \end{aligned}$$

In using logs. our calculations naturally end with a log., so that we must also be able to use the log. tables in the reverse way, i.e. read off the number whose log. we have obtained. For this purpose you have a table of "Antilogarithms," but many people prefer to use the "Logarithms" table, both for taking logs. and "delogarizing." It does not matter which you choose, and we will take a few illustrations of both methods.

EXAMPLE 8. Find antilog. 1·4487.

Looking at the log. table you will see that 4487 is in the " 1 " column opposite 28. The significant figures of the required number are therefore 281 and since the whole number part of the log. is 1 the antilog. is 28·1.

Antilog. 1·4487 = 28·1

Now look at the antilog. table. Take the line opposite 0·44 and trace it to the 8 column. The figures here are 2805. The figure in this line under the difference column for 7 is 4. Adding this, we get 2809. Since the log. is between 1 and 2, the number whose log. it is must be between 10 and 100, which tells us where to put the decimal point, i.e.—

Antilog. 1·4487 = 28·09

Sometimes you find very slight discrepancies between the results of using the log. table or the antilog. table, but obviously the difference between 28·09 and 28·1 is of no practical importance.

EXAMPLE 9. Find antilog. 0·0017.

Using the log. table first, 0017 cannot be found, but the significant figures will clearly be between 100 and 101 Looking at the differences columns we see 12 and 21 for 3 and 5 respectively. 17 is near enough half-way between 12 and 21, so we can take it as corresponding to 4. The significant figures therefore become 1004 and since the log has no whole number, the number whose log. it is will be between 1 and 10.

Therefore Antilog. 0·0017 = 1·004

The second method, using the antilog. tables, is as follows. Take the line opposite 0·00 and trace it to the 1 column. Figures here are 1002. Now continue to the difference column headed 7 and we find the figure 2. Add this and we get 1004. The position of the decimal point is decided in the same way, so that

Antilog. 0·0017 = 1·004

Here we get exactly the same result in each case, and with a little practice you will soon be able to decide which method you prefer.

EXAMPLE 10. Find antilog. $\bar{2}$·7777.

From log. table—

Antilog. $\bar{2}$·7777 = 0·05994

From antilog. table—

Antilog. $\bar{2}$·7777 = 0·05994

EXAMPLE 11. Find antilog. 5·6113.

From log. table—

Antilog. 5·6113 = 408,600

From antilog. table—

Antilog. 5·6113 = 408,600

How to Use Logs. Now we come to the real business. First of all, here are the facts. The most tiresome processes in arithmetic are multiplication, division, powers and roots. This is what the use of logs. does for us—

Multiplication becomes addition.
Division becomes subtraction.
Powers become multiplication.
Roots become division.

We will take first of all one simple example of each, so that you can see the trick.

EXAMPLE 12. Multiply 35·41 by 507.

For this calculation the process consists of three steps.

(a) Take the logs. of each number.
(b) Add the logs. together.
(c) Take the antilog. which will be the answer.

It is written out like this—

$$\begin{aligned} \text{Log. } (35{\cdot}41 \times 507) &= 1{\cdot}5491 + 2{\cdot}7050 \\ &= 4{\cdot}2541 \\ \text{Antilog. } 4{\cdot}2541 &= 17{,}950. \textit{ Answer.} \end{aligned}$$

Now let us compare the result with actual multiplication—

$$\begin{array}{r} 35{\cdot}41 \\ 507 \\ \hline 24787 \\ 177050 \\ \hline 17952{\cdot}87 \end{array}$$

So that the error introduced by using 4-figure log. tables is 2·87 in 17,950, or 0·016 per cent—quite negligible in practice.

EXAMPLE 13. Evaluate $\frac{12941{\cdot}6}{314{\cdot}7}$.

The three steps here are—

(a) Take the two logs.
(b) Subtract the log. of the divisor from the log. of the dividend.
(c) Take the antilog.

$$\begin{aligned} \text{Log. } \frac{12941{\cdot}6}{314{\cdot}7} &= 4{\cdot}1120 - 2{\cdot}4979 \\ &= 1{\cdot}6141 \\ \text{Antilog. } &= 41{\cdot}13. \textit{ Answer.} \end{aligned}$$

By arithmetic we shall get—

$$\begin{array}{l} 3147)129416(41{\cdot}123 \\ 12588 \\ \hline 3536 \\ 3147 \\ \hline 3890 \\ 3147 \\ \hline 7430 \\ 6294 \\ \hline 11360 \\ 9441 \\ \hline 1919 \end{array}$$

The error due to the use of logs. is 0·007 in 41·13, or 0·017 per cent.

EXAMPLE 14. Evaluate $(36{\cdot}4)^2$.

In calculating powers, the three steps are—

(a) Take log. of number.
(b) Multiply the log. by the power required.
(c) Take the antilog.

$$\begin{aligned} \text{Log. } (36{\cdot}4)^2 &= 2 \times 1{\cdot}5611 \\ &= 3{\cdot}1222 \\ \text{Antilog. } &= 1325. \textit{ Answer.} \end{aligned}$$

By arithmetic—

$$\begin{array}{r} 36{\cdot}4 \\ 36{\cdot}4 \\ \hline 1456 \\ 2184 \\ 1092 \\ \hline 1324{\cdot}96 \end{array}$$

The error is quite negligible.

EXAMPLE 15. Evaluate $\sqrt{318}$

In calculating roots, the three steps are—

(*a*) Take log. of number.
(*b*) Divide the log. by the root required.
(*c*) Take the antilog.

$$\text{Log. } \sqrt{318} = \frac{2{\cdot}5024}{2} = 1{\cdot}2512$$

$$\text{Antilog.} = 17{\cdot}83 \quad \textit{Answer.}$$

Testing this by squaring—

$$\begin{array}{r} 17{\cdot}83 \\ 17{\cdot}83 \\ \hline 5349 \\ 14264 \\ 12481 \\ 1783 \\ \hline 317{\cdot}9089 \end{array}$$

The error again is too small to matter in all ordinary work.

The examples just given are, of course, very simple and the time required to do them without the aid of logs. is very little more than with logs. Even so, the use of logs. shows some saving in time and is much less fatiguing mentally. In addition to these advantages, the use of logs. reduces the chances of error, provided the calculator is accustomed to them and can use them with confidence.

There are, of course, innumerable examples of calculations in which the use of logs. is imperative, because there is no other reasonable way of dealing with them.

Take the example quoted earlier in the chapter—

$$\frac{240}{36} \times 2{\cdot}7182^{-0{\cdot}048}$$

$2{\cdot}7182^{-0{\cdot}048}$ stumps us completely in arithmetic, but logs. make it easy, as you should now recognize. Here is the working—

$$\text{Log.}\left(\frac{240}{36} \times 2{\cdot}718^{-0{\cdot}048}\right)$$

$$= 2{\cdot}3802 - 1{\cdot}5563 + (0{\cdot}4343 \times -0{\cdot}048)$$

$$= 0{\cdot}8239 + (0{\cdot}4343 \times -0{\cdot}048)$$

We can use logs. again for the expression in brackets.

$$\begin{aligned}
\text{Log. } (0{\cdot}4343 \times 0{\cdot}048) &= \bar{1}{\cdot}6378 + \bar{2}{\cdot}6812 \\
&= \bar{2}{\cdot}3190 \\
\text{Antilog. } &= 0{\cdot}02084 \\
\text{Original log. } &= 0{\cdot}8239 - 0{\cdot}02084 \\
&= 0{\cdot}8030 \\
\text{Antilog. } &= 6{\cdot}353. \quad \textit{Answer.}
\end{aligned}$$

There are two points to observe here.

1. In calculating $(0{\cdot}4343 \times - 0{\cdot}048)$ we can ignore the $-$ sign, which does not affect the result numerically.

2. In adding the two logs. $\bar{1}{\cdot}6378$ and $\bar{2}{\cdot}6812$ the final whole number must be carefully dealt with.

$$\begin{array}{r}
\bar{1}{\cdot}6378 \\
\underline{\bar{2}{\cdot}6812} \\
\bar{2}{\cdot}3190
\end{array}$$

In the final step we add 6 and 6 and carry 1, making it 13. This gives 3 for the first decimal place. Now we add $\bar{1}$ and $\bar{2}$, giving $\bar{3}$, i.e. $- 3$, and we have $+ 1$ to carry, making $- 2$, i.e. $\bar{2}$.

Now that you have seen how logs. replace division by subtraction, it is worth while pausing for a moment to show why the logs. of numbers less than 1 are negative. For instance, take log. 0·2. We can deal with this as follows—

$$0{\cdot}2 = \frac{2}{10}$$

$$\begin{aligned}
\text{Log. } \frac{2}{10} &= \text{log. } 2 - \text{log. } 10 \\
&= 0{\cdot}3010 - 1
\end{aligned}$$

which of course is $- 0{\cdot}6990$.

As you have seen, however, we do not actually do the final subtraction, but keep the log. in the form $0{\cdot}3010 - 1$, i.e. $\bar{1}{\cdot}3010$. The $\bar{1}$ is negative, but the ·3010 is positive. It is important to remember this, or you may make mistakes, but practice in the use of logs. soon makes one quite familiar with the very few pitfalls there are.

There is one other snag which is encountered from time to time. Consider this case.

Example 16. Calculate $\sqrt{0{\cdot}712}$.

$$\text{Log. } \sqrt{0{\cdot}712} = \frac{\bar{1}{\cdot}8525}{2}$$

How are we to divide the log. by 2 when part of it is negative and part positive? The safest way is just to make the subtraction first, then divide,

and finally put the log. into the usual form by taking the next highest whole number and subtracting again.

$$\frac{\bar{1}\cdot8525}{2} = \frac{-1 + 0\cdot8525}{2}$$
$$= \frac{-0\cdot1475}{2} = -0\cdot0737$$

The actual log. is thus $-0\cdot0737$. The next whole number above this is 1, so that $-0\cdot0737 = \bar{1}\cdot9263$.

Antilog. $= 0\cdot844$. *Answer.*

EXAMPLE 17. Find $(0\cdot0095)^{\frac{1}{3}}$.

$$\text{Log. } (0\cdot0095)^{\frac{1}{3}} = \frac{\bar{3}\cdot9777}{3}$$
$$= \frac{-2\cdot0223}{3} = -0\cdot6741$$
$$= \bar{1}\cdot3259$$

Antilog. $= 0\cdot2118$. *Answer.*

EXAMPLE 18. Calculate $\sqrt{0\cdot00004}$.

$$\text{Log. } \sqrt{0\cdot00004} = \frac{\bar{5}\cdot6021}{2}$$
$$= \frac{-4\cdot3979}{2} = -2\cdot1989$$
$$= \bar{3}\cdot8011$$

Antilog. $= 0\cdot006326$. *Answer.*

The foregoing examples, if you have followed them thoroughly and checked them from the log. tables, should have convinced you how delightfully easy calculations become with the aid of logs. Write down a few awkward sums every day and wield the log. table on them, and at the end of a week no calculation you are likely to meet with on a radio course will give you a moment's anxiety. There is another big advantage too, which familiarity with logs. confers. Calculations are like railway journeys—they occur solely with the object of getting from one point to another, and in themselves are rather a nuisance. Without a knowledge of logs. your calculations will resemble the slowest of goods trains, and an accident on the way is highly probable. Master the use of logs. and you will travel first-class by crack express and always get there on time. Besides speed and accuracy, you will not get pre-occupied with the calculation, which so often happens with non-logarithmic students. It is a pity to get so involved in arithmetic and make such heavy going of the intermediate work as to lose sight of the result and its lesson.

Actual, Relative, and Percentage Error. No engineering calculations are absolutely accurate because no quantity can be measured with absolute accuracy. The floor of a room may be measured and found to be, say, 12 ft. $\times$ 8 ft.

Then $\qquad$ Area $= 12 \times 8 = 96$ sq. ft.

But owing to stretching of the tape measure or errors in observation, the true dimensions may be, say, 12 ft. 0·02 in. and 7 ft. 11·75 in.

The true area is therefore

$$\frac{144{\cdot}02 \times 95{\cdot}75}{144} \text{ sq. ft.}$$

$$\begin{aligned} \text{Log. area} &= 2{\cdot}1585 + 1{\cdot}9811 - 2{\cdot}1584 \\ &= 1{\cdot}9812 \\ \text{Area} &= 95{\cdot}77 \text{ sq. ft.} \end{aligned}$$

The actual error is 0·23 sq. ft.

The relative error is $\frac{0{\cdot}23}{95{\cdot}77} = 0{\cdot}0024$ or 24 parts in 10,000.

The percentage error is 0·24 per cent.

It follows that in practical work it is waste of time carrying calculation to a degree of accuracy far in excess of the accuracy of the data.

A well-known example from radio will emphasize this.

The velocity of ether waves is always taken as 3×10^8 metres per second, but this figure is only an approximation and is actually the nearest round number to the true velocity. If we wish to calculate the frequency of a wavelength of 46·2 metres, the nearest kc. is quite accurate enough, and to carry the calculation to the last significant figure is not only meaningless but may be inaccurate.

$$f = \frac{3 \times 10^8}{\lambda} = \frac{3 \times 10^8}{46{\cdot}2}$$

$$\begin{aligned} \text{Log. } f &= 8{\cdot}4771 - 1{\cdot}6646 \\ &= 6{\cdot}8125 \\ f &= 6{,}494{,}000 \text{ c.p.s.} \\ &= 6494 \text{ kcs.} \end{aligned}$$

An electrical example will be useful.

Calculate the impedance of a coil having an inductance of 180 μH. to a frequency of 1200 kcs.

$$2\pi fL = \frac{6{\cdot}28 \times 12 \times 1\cancel{0}^{5} \times 18\cancel{0}}{1\cancel{0}^{6}}$$

$$\begin{aligned} \text{Log. impedance} &= 0{\cdot}7980 + 2{\cdot}3345 \\ &= 3{\cdot}1325 \\ \text{Impedance} &= 1357 \text{ ohms} \end{aligned}$$

If we take this as 1350, or even 1300 or 1400, it will be quite accurate enough in most ordinary cases, because both the frequency and the inductance will often be a little different from their supposed values.

Both these examples show how the accuracy of logs. is ample

for practical work, and the slight additional accuracy obtained by pursuing pure arithmetical multiplication and division to the bitter end presents no advantage.

How to Ensure Accuracy in Working. No calculator is infallible and we all make slips at times. The thing is to avoid major errors. Everyone engaged in technical work should regard calculations in this way—an error should be viewed with surprise and some chagrin as an unusual occurrence. Never be satisfied to remain in the "Please, teacher, is this right?" stage. You should *know* whether your result is right or not, and, if in doubt, you should be prepared to check the working yourself and stand or fall accordingly. *The commonest of all faults in calculating is lack of method and slipshod ways of writing down the steps.*

This includes making unidentifiable calculations in odd corners of the paper, failing to distinguish between logs. and antilogs., omitting too many steps, mixing logarithmic work with purely arithmetical processes. Not only do such habits invite all kinds of errors, but it becomes practically impossible to check back and find the error.

As a final lesson in calculating, a number of examples will be fully worked out. Every step will be shown, perhaps rather more than really necessary, and a practised calculator would certainly omit some of them. But do not be in too great a hurry—it is better to take a minute or two longer over the job than spend a quarter of an hour finding a mistake.

Calculate slowly and carefully at first. The use of logs. will give you enormous speed compared to working without them, and when you can use logs. easily and confidently you will then be a thoroughly competent calculator.

EXAMPLE 19. Two resistances of 105 and 97 ohms respectively are connected in parallel across 440 volt D.C. mains. Find the watts taken.

$$\text{Total resistance} = \frac{105 \times 97}{202}$$

$$\begin{aligned}
\text{Watts} &= \frac{E^2}{R} = \frac{440^2 \times 202}{105 \times 97} \\
\text{Log. watts} &= 2 \times 2{\cdot}6435 + 2{\cdot}3054 - 2{\cdot}0212 - 1{\cdot}9868 \\
&= 5{\cdot}2870 + 2{\cdot}3054 - 4{\cdot}0080 \\
&= 7{\cdot}5924 - 4{\cdot}0080 \\
&= 3{\cdot}5844 \\
\text{Watts} &= 3841. \ \textit{Answer.}
\end{aligned}$$

EXAMPLE 20. Find resistance of 100 yds. of 20 S.W.G. hard-drawn copper wire.

The formula is $$R = \frac{sL}{A}$$

where R = resistance in ohms,
s = specific resistance per cm. cube,
L = length in cms.,
A = cross-section in sq. cms.

From an engineer's pocket-book we get—

$s = 1{\cdot}6$ microhms per cm. cube (for hard-drawn copper).

Diameter of 20 S.W.G. wire is 0·9 mm.

$$\text{Therefore} \quad A = \frac{\pi d^2}{4} = \frac{3{\cdot}14 \times (0{\cdot}09)^2}{4}$$
$$= 0{\cdot}785 \times 0{\cdot}0081$$
$$\text{Log. } A = \bar{1}{\cdot}8949 + \bar{3}{\cdot}9085 = \bar{3}{\cdot}8034$$
$$A = 0{\cdot}00636 \text{ sq. cms.}$$
$$L = 100 \text{ yds.} = 100 \times 36 \times 2{\cdot}54 \text{ cms.}$$
$$R = \frac{1{\cdot}6 \times 3600 \times 2{\cdot}54}{10^6 \times 0{\cdot}00636}$$
$$\text{Log. } R = 0{\cdot}2041 + 3{\cdot}5563 + 0{\cdot}4048 - 6{\cdot}00 - \bar{3}{\cdot}8034$$
$$= 4{\cdot}1652 - 3{\cdot}8034$$
$$= 0{\cdot}3618$$
$$R = 2{\cdot}3 \text{ ohms. } \textit{Answer.}$$

EXAMPLE 21. Find the natural frequency of a circuit consisting of a coil, inductance 75 μH. and a condenser 0·0002 mfd

$$f = \frac{1}{2\pi\sqrt{LC}}$$

where f = frequency in c.p.s.,
L = inductance in henries,
C = capacity in farads.

In this type of calculation, be very careful about converting the units to conform to the formula—many mistakes are made through failure to carry this out.

$$L = 75\mu\text{H.} = \frac{75}{10^6} \text{ henries}$$
$$C = 0{\cdot}0002 \text{ mfd.} = \frac{0{\cdot}0002}{10^6} \text{ farads}$$
$$f = \frac{1}{6{\cdot}28\sqrt{\frac{75}{10^6} \times \frac{0{\cdot}0002}{10^6}}}$$
$$= \frac{10^6}{6{\cdot}28\sqrt{75 \times 0{\cdot}0002}}$$
$$= \frac{10^6}{6{\cdot}28\sqrt{0{\cdot}0150}}$$
$$\text{Log. } f = 6{\cdot}00 - \{0{\cdot}7980 + (\tfrac{1}{2} \times \bar{2}{\cdot}1761)\}$$
$$= 6{\cdot}00 - \{0{\cdot}7980 + (\tfrac{1}{2} \times -1{\cdot}8239)\}$$
$$= 6{\cdot}00 - \{0{\cdot}7980 + (-0{\cdot}9119)\}$$
$$= 6{\cdot}00 - \{0{\cdot}7980 + \bar{1}{\cdot}0881\}$$
$$= 6{\cdot}00 - \bar{1}{\cdot}8861$$
$$= 6{\cdot}1139$$
$$f = 1{,}300{,}000$$
$$= 1300 \text{ kcs.}$$

EXAMPLE 22. The anode current in a certain valve is known to follow the law I (milliamps) $= kV^{\frac{3}{2}}$. Find the current for $V = 50$ volts where k is taken as 0·0016.

$$I = 0{\cdot}0016 \times (50)^{\frac{3}{2}}$$

$50^{\frac{3}{2}}$ is the same as $\sqrt{50^3} = \sqrt{125{,}000}$.

$$\begin{aligned} I &= 0{\cdot}0016 \times \sqrt{125{,}000} \\ \text{Log. } I &= \bar{3}{\cdot}2041 + \tfrac{1}{2} \times 5{\cdot}0969 \\ &= \bar{3}{\cdot}2041 + 2{\cdot}5484 \\ &= \bar{1}{\cdot}7525 \\ I &= 0{\cdot}565 \text{ mA. } \textit{Answer.} \end{aligned}$$

EXAMPLE 23. A tuned anode circuit consists of an inductance, value 60 μH., a condenser of 0·0001 mfd., and has a total resistance of 15 ohms. Find its impedance to the resonant frequency.

$$\text{Impedance} = \frac{L}{CR} \text{ ohms}$$

where L = inductance in henries,
C = capacity in farads,
R = resistance in ohms

$$\text{Impedance} = \frac{L}{CR} = \frac{60 \times \cancel{10^6}}{\cancel{10^6} \times 0{\cdot}0001 \times 15}$$
$$= \frac{4}{0{\cdot}0001}$$
$$= 40{,}000 \text{ ohms. } \textit{Answer.}$$

Logarithmic working is clearly superfluous in this example.

EXAMPLE 24. A circuit has a resistance of 55 ohms, and an inductance of 75 henries. If 110 volts is applied, find the current $\frac{1}{500}$th second after closing the switch.

The formula is $$I = \frac{E}{R}\left(1 - \varepsilon^{\frac{-Rt}{L}}\right)$$

where I = current in amperes,
E = e.m.f. in volts,
R = resistance in ohms,
L = inductance in henries,
t = time in seconds,
ε = 2·7182.

The formula does not allow of immediate logarithmic work, because it contains a subtraction. It must therefore be dealt with in stages, as follows—

$$\frac{Rt}{L} = \frac{55 \times 0{\cdot}002}{75}$$

$$\begin{aligned} \text{Log. } \frac{Rt}{L} &= 1{\cdot}7404 + \bar{3}{\cdot}3010 - 1{\cdot}8751 \\ &= \bar{1}{\cdot}0414 - 1{\cdot}8751 \\ &= \bar{3}{\cdot}1663 \\ \frac{Rt}{L} &= 0{\cdot}00146 \\ \varepsilon^{\frac{-Rt}{L}} &= 2{\cdot}7182^{-0{\cdot}00146} \\ \text{Log } \varepsilon^{\frac{-Rt}{L}} &= -\,0{\cdot}00146 \times 0{\cdot}4343 \end{aligned}$$

$$\left\{\begin{aligned}\text{Log. } (0{\cdot}00146 \times 0{\cdot}4343) &= \bar{3}{\cdot}1663 + \bar{1}{\cdot}6378 \\ &= \bar{4}{\cdot}8041 \\ \text{Antilog.} &= 0{\cdot}000637\end{aligned}\right\}$$

$$\text{Log. } \varepsilon^{\frac{-Rt}{L}} = \bar{1}{\cdot}999363$$

$$\varepsilon^{\frac{-Rt}{L}} = 0{\cdot}9985$$

$$(1 - \varepsilon^{\frac{-Rt}{L}}) = 1 - 0{\cdot}9985 = 0{\cdot}0015$$

Therefore

$$I = \frac{110}{55}(0{\cdot}0015)$$

$$= 0{\cdot}0030 \text{ amp. } \textit{Answer.}$$

EXERCISES II

Evaluate the following expressions—

1. $6{\cdot}28 \times 400 \times 72{\cdot}3.$
2. $3{\cdot}14 \times (6{\cdot}72)^2 \times 18{\cdot}1.$
3. $\sqrt[3]{0{\cdot}0008} \times 2{\cdot}718^{-1{\cdot}2}$
4. $\dfrac{10^6}{(3{\cdot}818)^2}.$
5. $\sqrt{(896)^2 + \left\{\left(\dfrac{6{\cdot}28 \times 3 \times 10^5 \times 232}{10^6}\right) - \left(\dfrac{10^6}{6{\cdot}28 \times 3 \times 10^5 \times 0{\cdot}004}\right)\right\}^2}$
6. $\dfrac{220}{42}\left\{2{\cdot}7182^{0{\cdot}03}\right\}$
7. $64 \times 2{\cdot}718^{(0{\cdot}06 \times 2{\cdot}3\pi)}$

8. Two resistances of 35·8 and 46·6 ohms are connected in parallel across 198 volts D.C. Find the current flowing and the watts taken from the supply.

9. The natural frequency of an oscillatory circuit is given by the formula—

$$f = \frac{1}{2\pi\sqrt{LC}}$$

where f = frequency in cycles per second
L = inductance in henries
C = capacity in farads.

Calculate the frequency in kcs of a circuit having an inductance of 130 mics. and a capacity of 0·0004 mfd.

10. The stage gain of an amplifier is given by the formula—

$$\text{Stage gain} = \frac{\mu R_A}{R_O + R_A}$$

where μ = valve amplification ratio
R_A = anode circuit impedance
R_O = valve impedance.

Calculate the stage gain when $\mu = 225$, $R_O = 150{,}000$ ohms, $R_A = 230{,}000$ ohms.

11. Find the impedance of a circuit having a resistance of 48 ohms and an inductance of 28 mics. to a frequency of 3·5 megacycles, given that

$$\text{Impedance} = \sqrt{R^2 + (\omega L)^2}$$

where R = resistance in ohms
L = inductance in henries
$\omega = 2\pi \times$ frequency in c.p.s.

12. Calculate the resistance of 2 miles of 16 S.W.G. round copper wire.

$$R = \frac{sL}{A}$$

where R = resistance in ohms
L = length in centimetres
A = cross-sectional area in square centimetres.
s = specific resistance per cm. cube. (Take the value of s as 1·7 microhms.)

Diameter of 16 S.W.G. wire = 1·6 mm.

CHAPTER III

ALGEBRA

In algebra, symbols are used to denote quantities. These symbols are letters of the English or Greek alphabet. The use of such symbols enables mathematicians to carry out mathematical reasoning and obtain results, and also makes possible the construction of formulae. For example, Ohm's Law is stated as

$$I = \frac{E}{R}$$

That is algebra. I denotes the current in amperes, E the e.m.f. in volts, and R the resistance in ohms. It is a short and convenient way of stating the relationship between two or more quantities. The actual symbols used are quite immaterial so long as we know what they represent. The area of a rectangular figure is length times breadth, so we could put this down as a formula.

If A = area in square feet
x = length in feet
y = breadth in feet

$$A = xy$$

Note particularly that a formula is entirely meaningless and useless, unless we know—

(*a*) What the symbols denote.
(*b*) The units in which they are to be expressed.

From the wireless operator's point of view, the formula is the most immediate application of algebra, but it is a good thing to have some idea of the elementary rules of algebraic processes, which are quite simple.

Addition. The sum of x and x is $x + x = 2x$. Similarly $x + x + x = 3x$.

The sum of x and y is $x + y$. No further step can be taken unless we know the numerical values of x and y, when the algebra, of course, becomes arithmetic.

Subtraction. The following statements are self-evident—

$$x - x = 0$$
$$2x - x = x$$
$$9x - 4x = 5x$$

To subtract y from x gives the answer $x - y$.

Again, no further step can be made algebraically.

Multiplication. When two quantities expressed by symbols are to be multiplied they are written adjacent to each other.

Thus,
$$a \times b = ab$$
$$x \times y = xy$$
$$p \times q \times r = pqr$$

Division. The division of one algebraic symbol by another is written as a fraction. x divided by $y = \frac{x}{y}$.

The sum of a and b divided by $p = \frac{a + b}{p}$.

Powers. These are indicated as explained in Chapter II.

Thus,
$$x \times x = x^2$$
$$x \times x \times x = x^3$$
and so on.

Roots. Roots are also indicated as previously shown.

The Square root of $x = \sqrt{x}$ or $x^{\frac{1}{2}}$

Cube root of $a = \sqrt[3]{a}$ or $a^{\frac{1}{3}}$

Fourth root of $y = \sqrt[4]{y}$ or $y^{\frac{1}{4}}$

Brackets. Round, curly, and square brackets may be used in algebraic expressions. The same rules apply as regards signs, and multiplication or division may be indicated. Here are some simple examples.

EXAMPLE 1.
$$\begin{aligned} & 4a + (a - 2b) \\ &= 4a + a - 2b \\ &= 5a - 2b \end{aligned}$$

EXAMPLE 2.
$$\begin{aligned} & 9x - (y + 8x) \\ &= 9x - y - 8x \\ &= x - y \end{aligned}$$

EXAMPLE 3.
$$\begin{aligned} & a + b(c - d) \\ &= a + bc - bd \end{aligned}$$

EXAMPLE 4.
$$\begin{aligned} & 12a - \{3b + a(a + 3)\} \\ &= 12a - \{3b + a^2 + 3a\} \\ &= 12a - 3b - a^2 - 3a \\ &= 9a - 3b - a^2 \end{aligned}$$

EXAMPLE 5.
$$\begin{aligned} & 10x^2 + 2[x^2 + xy - \tfrac{1}{2}\{4x^2 + 3(2xy - 1)\}] \\ &= 10x^2 + 2[x^2 + xy - \tfrac{1}{2}\{4x^2 + 6xy - 3\}] \\ &= 10x^2 + 2[x^2 + xy - 2x^2 - 3xy + \tfrac{3}{2}] \\ &= 10x^2 + 2x^2 + 2xy - 4x^2 - 6xy + 3 \\ &= 8x^2 - 4xy + 3 \end{aligned}$$

Factorization. It is useful to know some of the more common results of algebraic multiplication and the method of factorizing simple expressions.

(1) *Square of a Binomial.* Binomial simply means the sum or difference of two quantities. Thus $(a + b)$ or $(3x - 4y)$ are binomials.

The square of a binomial is the sum of the squares of each term plus twice their product.

Suppose we wanted to square $2 + 3$ keeping the 2 and 3 separate. The working would be as follows—

$$\begin{aligned}(2 + 3)^2 &= (2 + 3) \times (2 + 3)\\ &= 2(2 + 3) + 3(2 + 3)\\ &= 2^2 + (2 \times 3) + (2 \times 3) + 3^2\\ &= 2^2 + 2(2 \times 3) + 3^2\end{aligned}$$

Now compare the algebraic process.

$$\begin{aligned}(a + b)^2 &= (a + b) \times (a + b)\\ &= a(a + b) + b(a + b)\\ &= a^2 + ab + ab + b^2\\ &= a^2 + 2ab + b^2\end{aligned}$$

Similarly,

$$\begin{aligned}(3x - 4y)^2 &= 9x^2 - 24xy + 16y^2\\ (2p + q)^2 &= 4p^2 + 4pq + q^2\\ (x^2 - y)^2 &= x^4 - 2x^2y + y^2\end{aligned}$$

Note that the sign of the middle term in the square is positive or negative according to whether the binomial is a sum or a difference. Also that the third term is always positive because the square of a negative quantity is positive.

(2) *Difference of Two Squares.* Consider the expression

$$(3 + 2)\,(3 - 2)$$

This is equal to $3(3 + 2) - 2(3 + 2)$

$$\begin{aligned}&= 3^2 + (3 \times 2) - (3 \times 2) - 2^2\\ &= 3^2 - 2^2\end{aligned}$$

Similarly, $(a + b)\,(a - b)$

$$\begin{aligned}&= a(a + b) - b(a + b)\\ &= a^2 + ab - ab - b^2\\ &= a^2 - b^2\end{aligned}$$

Thus the difference of two squares is equal to the product of the sum and difference of their square roots.

For example, $9x^2 - 4y^2$

$$= (3x + 2y)(3x - 2y)$$

Note that only the *difference* of two squares can be factorized. The *sum* of two squares does not allow any such treatment. $a^2 + b^2$ cannot be boiled down any further.

(3) *Product of Two Binomials.* Let us multiply out

$$(2a + 3b)(a + b)$$

$$\begin{array}{l} 2a + 3b \\ a + b \\ \hline 2a^2 + 3ab \\ + 2ab + 3b^2 \\ \hline 2a^2 + 5ab + 3b^2 \end{array}$$

This can always be done without writing it out in full. It is only necessary to multiply each term in one pair of brackets by each term in the other and combine any similar terms.

$$(2a + 3b)(a + b)$$

Inspecting the above, we see at once that the "end" terms will be—

$$2a^2 \qquad\qquad + 3b^2$$

and the other two will be $+ 2ab$ and $+ 3ab$, giving $+ 5ab$, so that the result is

$$2a^2 + 5ab + 3b^2$$

Take another example of this.

$$(x - 2y)(2x + y)$$

We get $2x^2 + xy - 4xy - 2y^2$

$$= 2x^2 - 3xy - 2y^2$$

Now consider briefly the reverse process, which is called Factorization. Suppose we wish to split up the expression

$$2a^2 + 5ab + 3b^2$$

into two binomial factors.

It is very easy, as you will see from the following reasoning.

First of all put down the brackets—

$$(\qquad)(\qquad)$$

Now $2a^2$ can only be $2a \times a$; there is no other way of splitting it. So we have

$$(2a \qquad)(a \qquad)$$

Similarly, $3b^2$ can only be divided into $3b \times b$, but which bracket will have $3b$ and which one b? The middle term tells you this. To get $5ab$ we must have $3ab + 2ab$, so that the answer is—

$$(2a + 3b)(a + b)$$

If we had put $(2a + b)(a + 3b)$

we should get $2a^2 + 7ab + 3b^2$

which is wrong.

Similarly,

$$2a^2 - 5ab + 3b^2$$
$$= (2a - 3b)(a - b)$$
$$2a^2 + 5ab - 3b^2$$
$$= (2a - b)(a + 3b)$$
$$2a^2 - 5ab - 3b^2$$
$$= (2a + b)(a - 3b)$$

Simple Equations. An equation is a statement in symbolic form of the relationship existing between *two or more* quantities. Solving the equation means finding the value of one of these quantities when all the others are known.

For example, $I = \frac{E}{R}$ is a simple equation, and knowing any two of the quantities we can find the third. The equation can, of course, be re-arranged in the forms

$$E = IR$$

$$R = \frac{E}{I}$$

The unknown quantity is always brought over to the left-hand side of the equation in the process of solution.

Certain simple rules apply to all equations.

(1) The same quantity may be added to or subtracted from both sides of the equation.

(2) Both sides may be multiplied or divided by any quantity.

(3) Any term may be transferred from one side to the other, provided its sign is changed.

A simple example will illustrate these three rules.

Suppose $x = 2$

Then, obviously $x + 1 = 2 + 1$

or $x - 1 = 2 - 1$

or $x + (a - b) = 2 + (a - b)$

Taking Rule 2,

$$x = 2$$

Then

$$2x = 2 \times 2$$
$$xy = 2y$$
$$\frac{x}{3} = \frac{2}{3}$$
$$\frac{x}{p} = \frac{2}{p}$$
$$x^3 = 2^3$$
$$\sqrt{x} = \sqrt{2}$$

Taking Rule 3,

$$x = 2$$
$$x - 2 = 0$$
$$2 - x = 0$$

This is really the same as subtracting either x or 2 from each side, and when a term is transferred from one side to the other, it is actually a subtraction that is being carried out, unless the sign is $-$ before the transference, in which case the process is equivalent to addition.

Thus, suppose $x - 3 = 5$. Add 3 to each side, and we get $x = 5 + 3$, which is the same as taking the -3 over to the right and changing its sign.

Any formula is, of course, an equation, and can be re-arranged to obtain the unknown quantity. We will take one or two examples from radio and consider how to manipulate them.

EXAMPLE 6.

f = frequency in kcs ,
λ = wavelength in metres

$$f = \frac{3 \times 10^5}{\lambda}$$

If we now wish to find an expression for λ, the process of "cross-multiplication" is used. This is very simple, and is best illustrated as shown below—

$$\frac{a}{b} = \frac{c}{d}$$

That is

$$a \times d = b \times c$$

Going back to the example, we have

$$\frac{f}{1} = \frac{3 \times 10^5}{\lambda}$$

so that

$$f \times \lambda = 3 \times 10^5$$

Now divide both sides of the equation by f.

$$\frac{\cancel{f} \times \lambda}{\cancel{f}} = \frac{3 \times 10^5}{f}$$
$$\lambda = \frac{3 \times 10^5}{f}$$

EXAMPLE 7.

R = total resistance,

R_1, R_2 = values of two resistances in parallel.

$$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$$
$$=\frac{R_2}{R_1R_2}+\frac{R_1}{R_1R_2}$$
$$=\frac{R_2+R_1}{R_1R_2}$$

Therefore $$R=\frac{R_1R_2}{R_1+R_2}$$

Note in this example that when both sides of the equation are fractions, both sides can be inverted without upsetting its truth.

EXAMPLE 8.

f = frequency in cycles per second,

L = inductance in henries,

C = capacity in farads.

$$f=\frac{1}{2\pi\sqrt{LC}}$$

Suppose we know f and C and want to find L

$$f=\frac{1}{2\pi\sqrt{LC}}$$

Cross-multiply, and we get

$$2\pi f\sqrt{LC}=1$$

We cannot deal with L separately unless it is taken from under the root, so both sides of the equation must be squared.

$$4\pi^2f^2LC=1$$
$$L=\frac{1}{4\pi^2f^2C}$$

EXAMPLE 9

R_O = valve impedance,

R_A = anode circuit impedance,

μ = amplification factor of valve,

x = stage gain.

$$x=\frac{\mu R_A}{R_O+R_A}$$

Suppose we wish to find μ in terms of x, R_O, and R_A

Cross-multiplying, $x(R_O+R_A)=\mu R_A$

$$\mu=\frac{x(R_O+R_A)}{R_A}$$

EXAMPLE 10. The current taken by a low-resistance rejector circuit from a resonant voltage is $\frac{ERC}{L}$. What is its impedance?

In any circuit, $$\text{Current}=\frac{\text{voltage}}{\text{impedance}}$$

i.e. $$I=\frac{E}{Z}$$

where Z denotes impedance.

Therefore,
$$Z = \frac{E}{I}$$
$$= \frac{E}{\frac{ERC}{L}}$$
$$= E \times \frac{L}{ERC}$$
$$= \frac{L}{CR} \text{ ohms.}$$

EXAMPLE 11.

I = current in amperes,
E = e.m.f. in volts,
f = frequency in c.p.s.,
R = resistance in ohms,
L = inductance in henries.

Then for a series circuit
$$I = \frac{E}{\sqrt{R^2 + (2\pi fL)^2}}$$

Suppose f is the unknown quantity. Re-arrangement would be as follows.
$$I = \frac{E}{\sqrt{R^2 + (2\pi fL)^2}}$$
$$I\sqrt{R^2 + (2\pi fL)^2} = E$$
$$R^2 + 4\pi^2 f^2 L^2 = \frac{E^2}{I^2}$$
$$4\pi^2 f^2 L^2 = \frac{E^2}{I^2} - R^2$$
$$= \frac{E^2 - I^2R^2}{I^2}$$
$$f^2 = \frac{E^2 - I^2R^2}{4\pi^2 L^2 I^2}$$
$$f = \frac{\sqrt{E^2 - I^2R^2}}{2\pi LI}$$
or
$$f = \frac{\sqrt{(E + IR)(E - IR)}}{2\pi LI} \quad \text{(see Page 29 (2)).}$$

The latter form might be more convenient for logarithmic working.

The foregoing is an introduction to such algebraic processes as the wireless operator should know. As will be seen, it is largely a matter of handling expressions in accordance with a few simple rules. Regarded in this way, and used for this purpose, algebra is little more than a form of shorthand, and the use of symbols should not occasion any apprehension on the part of the student that a mathematical flood is about to engulf him.

EXERCISES III

1. Simplify the expression
$$15b - a + 5\{a + 2b + 3(a - 2b)\}$$

2. Square the following expressions—

(i) $2a + 3b$
(ii) $x - 5y$
(iii) $2p - q^2$
(iv) $p^2 + q^2$

3. Factorize the following expressions—

(i) $9a^2 - 81b^2$
(ii) $2a^2 - 7ab - 4b^2$
(iii) $2x^2 + 5xy + 3y^2$
(iv) $3p^4 - 2p^2q^2 - q^4$

4. If $\lambda = 1885\sqrt{LC}$ where λ = wavelength in metres, L = inductance in microhenries and C = capacity in mfds.

(i) Express C in terms of L and λ
(ii) Calculate C for $L = 12$ mics , $\lambda = 31$ metres.

5. The capacity of a parallel-plate condenser is given by the formula

$$C = \frac{AKn}{1{\cdot}131d \times 10^7}$$

where C = capacity in mfds.
A = area of each plate in sq. cms.
n = number of plates less one.
d = distance between plates in cms.
K = specific inductive capacity of dielectric.

(i) Express n in terms of the other variables.
(ii) Calculate no. of plates for the following values—

$C = 0{\cdot}0005$ mfd.
$A = 9$ sq. cms.
$d = 0{\cdot}5$ millimetre
$K = 2$.

6 $$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2}$$

where L_T = total inductance
$\left.\begin{matrix} L_1 \\ L_2 \end{matrix}\right\}$ = values of two inductances in parallel.

(i) Express L_1 in terms of L_T and L_2.
(ii) Calculate L_1 when $L_2 = 143$ mics., $L_T = 95$ mics.

CHAPTER IV

GEOMETRY AND TRIGONOMETRY

GEOMETRY is a branch of mathematics that no one can entirely escape. If you wish to go to a shop on the other side of the street you would normally make a "bee-line" for it, i.e. you would take the shortest route, which is the straight line joining the shop doorway to the point at which you make your decision. If you play billiards you are mentally estimating angles with almost every shot, and even in such a homely action as slicing bread you would judge parallelism of the knife with the edge of the loaf.

Plane geometry deals with the properties and relationships of plane figures—that is to say, figures drawn on a flat surface, which for theoretical purposes is assumed to be perfectly smooth and level. Formal books on geometry contain a large number of "Theorems" which start off with a statement regarding some figure or figures, the statement then being proved by a process of logical reasoning. This, of course, is the proper way to learn geometry and constitutes one of the best of all mental exercises. For the present purpose, however, there is no time for such formalities, and we must be content with picking out the important results which you really need for a radio course. These are quite simple and most interesting, and should they give you the desire to learn more of this fascinating subject, so much the better.

DEFINITIONS

Point. A geometrical point has position but no magnitude. (As it would be invisible if it had no magnitude, we have to compromise by making a small dot in the desired position. As a rule, however, an isolated point has no geometrical interest, a point being usually defined by the termination of a line or the intersection of two lines.)

Line. A geometrical line is terminated by two points and has length but no breadth.

(Here, again, it must have some breadth in order to be visible on the paper.)

Area. An area is the space enclosed by a series of lines, straight or curved. It has length and breadth but no thickness, and is called a two-dimensional figure.

Solid. A solid is a figure which has length, breadth, and thickness. This is called a three-dimensional figure.

Angle. An angle is formed by the intersection of two straight lines.

Triangle. A triangle is the figure enclosed by three straight lines which do not intersect at a point.

Circle. A circle is the path traced out by a point which moves at a

constant distance from a fixed point. The fixed point is called the centre of the circle and the path of the moving point is called its circumference. The distance from the centre to any point on the circumference is the radius of the circle.

Measurement of Angles. For most practical purposes "sexagesimal measure" is used.

The circle is divided into 360 degrees, and each degree is subdivided as follows—

$$1 \text{ degree} = 60 \text{ minutes}$$
$$1 \text{ minute} = 60 \text{ seconds}$$

In Fig. 1 a circle is shown with two lines drawn through its centre and terminated by its circumference. These lines are called diameters. If they divide the circle into four quarters, each angle at the point O will be $\frac{360°}{4} = 90°$. This is known as a right angle.

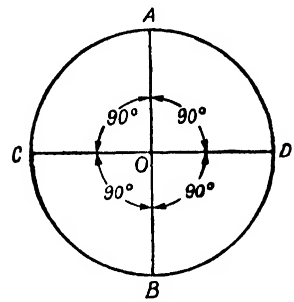

FIG. 1

In Fig. 2 are shown the various general types of angles.

(*a*) Any angle *less* than a right angle is called an *acute angle.*

(*b*) An angle of 90° is a *right angle.*

(*c*) An angle greater than 90° but less than 180° is called an *obtuse angle.*

(*d*) An angle of 180° is formed at any point in a straight line and is referred to as a *straight angle.*

(*e*) An angle of more than 180° is called a *reflex angle.*

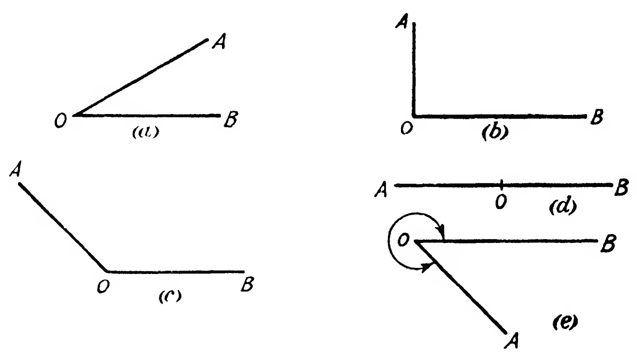

FIG. 2

Naming of Angles. An angle is usually specified by means of three letters, the centre one being its vertex, i.e. the point at which the angle is formed. Thus in Fig. 1 there are four angles, *AOD*, *DOB*, *BOC*, and *AOC*.

If only one angle is concerned, and there can be no doubt as to the meaning, one letter may be used. Thus in Fig. 2 (*a*) we could call the angle O instead of AOB or BOA. The recognized abbreviation for "angle" is $\angle$.

Two angles whose sum is 90° are said to be *complementary*.

Two angles whose sum is 180° are said to be *supplementary*.

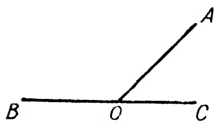

FIG. 3

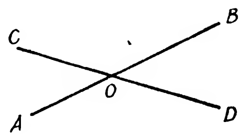

FIG. 4

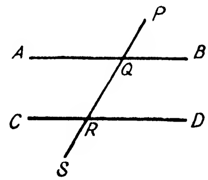

FIG. 5

In Fig. 3 the two angles AOB and AOC must obviously add up to 180° and are therefore supplementary, no matter what position the line AO occupies.

When two straight lines intersect, four angles are formed.

In Fig. 4 the angles COA and BOD are called vertically opposite angles and are always equal. Similarly the angles BOC and AOD are equal.

Parallels. Two straight lines are said to be parallel when they are the same distance apart throughout their length.

If another line is drawn cutting both of them, some important results arise.

In Fig. 5 AB and CD are parallel straight lines. $PQRS$ is called a transversal. Here are the results, which apply to any position of $PQRS$.

(1)
$$\angle AQR = \angle QRD$$
$$\angle BQR = \angle QRC$$

These pairs are called alternate angles.

(2)
$$\angle PQB = \angle QRD$$
$$\angle PQA = \angle QRC$$
$$\angle CRS = \angle AQR$$
$$\angle DRS = \angle BQR$$

These pairs are called exterior and interior opposite angles.

(3)
$$\angle AQR + \angle QRC = 180°$$
$$\angle BQR + \angle QRD = 180°$$

The two pairs of interior angles are therefore supplementary.

Triangles. Triangles are classified as follows—

(*a*) An *equilateral* triangle has all its sides of the same length.

(*b*) An *isosceles* triangle has two sides equal.

(*c*) A *scalene* triangle has all sides of different lengths.

(*d*) A *right-angled* triangle has one of its angles 90°.
(*e*) An *obtuse-angled* triangle has one angle greater than 90°.
The five types are shown in Fig. 6.

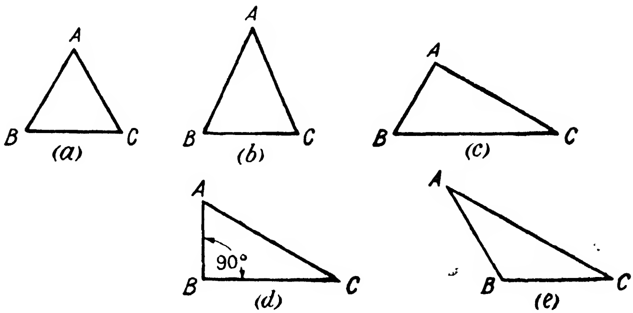

FIG. 6

Angles of a Triangle. The three angles of any triangle always add up to two right angles, i.e. 180°.

From this it follows that no triangle can have more than one right angle.

In an equilateral triangle, all sides and all angles are equal. Each angle is therefore $\frac{180°}{3} = 60°$.

In an isosceles triangle, the angles opposite the equal sides are equal. In Fig. 6 (*b*), $AB = AC$ and $\angle ABC = \angle ACB$.

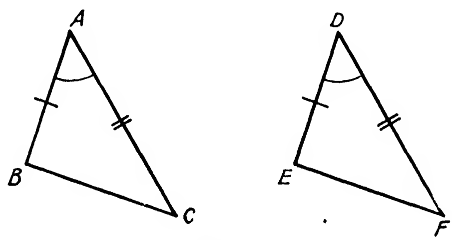

FIG. 7

Equality of Triangles. Two triangles are exactly equal under the following conditions—

(1) If two sides and the included angle are equal.

In Fig. 7, if

$$AB = DE$$
$$AC = DF$$
$$\angle BAC = \angle EDF$$

then the triangles are equal.

(2) If one side and two angles are equal.

In Fig. 8, if

$$AB = DE$$
$$\angle ABC = \angle DEF$$
$$\angle ACB = \angle DFE$$

then the triangles are equal.

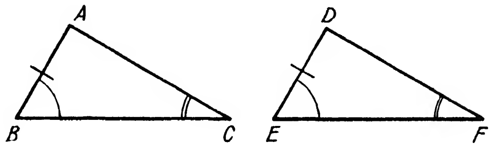

FIG. 8

(3) If the three sides are equal.

In Fig. 9, if

$$AB = DE$$
$$BC = EF$$
$$AC = DF$$

then the triangles are equal.

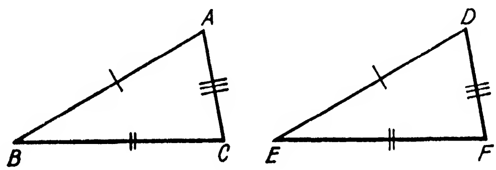

FIG. 9

Similar Triangles. If two triangles have all their corresponding angles equal they are said to be similar.

In Fig. 10, if PQ is parallel to BC, the two triangles APQ and ABC are equiangular and therefore similar.

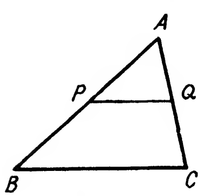

FIG. 10

In similar triangles the sides are proportional

That is to say, $\dfrac{AP}{AB} = \dfrac{AQ}{AC} = \dfrac{PQ}{BC}$.

If P and Q are the mid-points of AB and AC, $\dfrac{AP}{AB} = \dfrac{1}{2} = \dfrac{AQ}{AC}$, and it would be found that PQ is $\frac{1}{2}BC$.

The Right-angled Triangle. This is one of the most important of all geometrical figures in engineering. It has a special relationship between its sides, which constitutes the most famous theorem in geometry, called Pythagoras's theorem. Here it is.

Pythagoras's Theorem. The square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the other two sides.

In Fig. 11 ABC is a right-angled triangle, C being the right angle. The side AB opposite the right angle is called the hypotenuse.

By Pythagoras's theorem

$$AB^2 = AC^2 + BC^2$$

To test this simply, draw an angle of 90° as accurately as possible with a protractor and make its arms 4 inches and 3 inches respectively. Join up the third side and you will find it is exactly 5 inches long.

$$5^2 = 4^2 + 3^2$$

The relationship is true for any right-angled triangle.

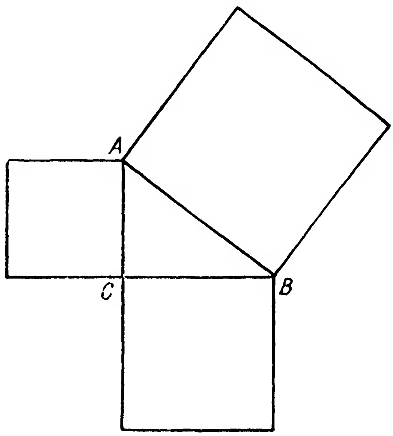

FIG. 11

Knowing the lengths of any two sides of a right-angled triangle, the third side can at once be calculated.

Suppose in Fig. 11, $AC = 2$ in. and $BC = 4{\cdot}5$ in.

Then
$$\begin{aligned} AB &= \sqrt{2^2 + (4{\cdot}5)^2} \\ &= \sqrt{4 + \frac{81}{4}} \\ &= \sqrt{\frac{97}{4}} \\ &= 4{\cdot}92 \text{ in.} \end{aligned}$$

Suppose
$$\begin{aligned} AB &= 6 \text{ in.} \\ AC &= 2 \text{ in.} \end{aligned}$$

Then
$$\begin{aligned} BC &= \sqrt{6^2 - 2^2} \\ &= \sqrt{(6 + 2)(6 - 2)} \\ &= \sqrt{32} \\ &= 5{\cdot}65 \text{ in.} \end{aligned}$$

The Parallelogram. A parallelogram is a four-sided figure whose opposite sides are parallel. It possesses the following properties—

(1) Opposite sides are equal.

(2) Opposite angles are equal.

(3) Each diagonal bisects the parallelogram, i.e. divides it into two equal triangles.

(4) The diagonals bisect one another.

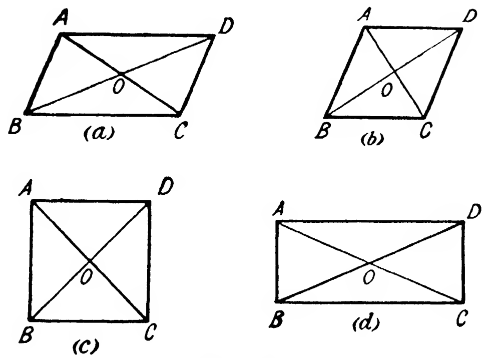

FIG. 12

Three special cases of parallelograms are as follows—

(1) If all sides are equal, but the angles not right angles, it is called a *rhombus*.

(2) If all sides are equal and all angles right angles, it is a *square*.

(3) If all angles are right angles, but adjacent sides not equal, it is a *rectangle*.

In Fig. 12 are shown (*a*) a parallelogram, (*b*) a rhombus, (*c*) a square, and (*d*) a rectangle.

In all cases—

$$AB = CD$$
$$AD = BC$$
$$AO = OC$$
$$BO = OD$$
$$\angle BAD = \angle BCD$$
$$\angle ABC = \angle ADC$$

In cases (*b*) and (*c*)—

$\angle$s at O are all 90°.

$\triangle$s AOB, AOD, COD, COB are all equal.

In cases (*c*) and (*d*)—

$$AC = BD$$

The Circle. A circle has already been defined as the locus (i.e. the path traced out) of a point moving at a constant distance from a fixed point—the centre.

A line drawn through the centre and terminated by the circumference divides the circle into two equal halves called semicircles, and this line is a diameter. The diameter of a circle is, of course, twice the radius.

There is a fixed relationship between the circumference of any circle and its diameter.

This is denoted by the Greek letter π (pronounced "pie"), and is actually an "incommensurable" number, i.e. it cannot be finally

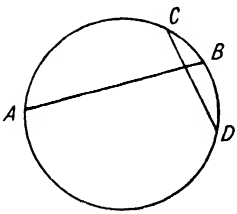

FIG. 13

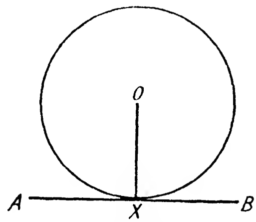

FIG. 14

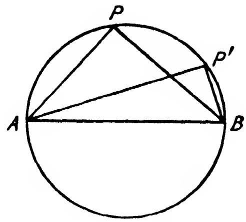

FIG. 15

calculated. The value of π is 3·14159 to five decimal places. For nearly all practical purposes 3·14 is near enough. (It is worth memorizing log. π, which is 0·4969.)

The following results should be part of your mental stock-in-trade—

If

$$r = \text{radius of } \bigcirc$$

$$d = \text{diameter of } \odot$$

$$d = 2r$$

$$\text{Circumference} = \pi d$$

$$\text{Area of circle} = \pi r^2 \text{ or } \frac{\pi d^2}{4}$$

Chord. A chord of a circle is a line terminated by the circumference, but not passing through the centre.

In Fig. 13, AB and CD are chords.

Tangent. A tangent to a circle is a line which touches but does not cut the circumference. A tangent makes an angle of 90° with the radius drawn to the point of contact.

In Fig. 14, AB is a tangent to the circle, touching the circumference at X. OX is a radius and the angles OXA and OXB will be 90°.

Angle in a Semicircle. The angle in a semicircle is 90°. In Fig. 15, AB is a diameter of the circle. Taking any point P on the circumference and joining PA and PB, it will be found that the angle so formed is always 90°. Thus in the figure, $APB = 90°$, $AP'B = 90°$.

TRIGONOMETRY

If you look in your pocket-book, you will probably find that after the log. and antilog. tables there is a table of "Trigonometric Functions of Angles." Every angle has six such functions and they are in constant use for engineering purposes.

Take any angle X (Fig. 16) and any point A on one of its arms. Drop the perpendicular AB, i.e. make ABX a right angle. It will

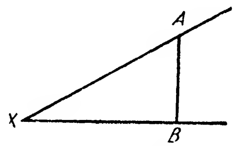

FIG. 16

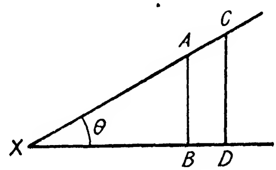

FIG. 17

be found that the following ratios are always the same for any position of the perpendicular AB.

$$\frac{AB}{AX} \qquad \frac{BX}{AX} \qquad \frac{AB}{BX}$$

To make this clear, look at Fig. 17, in which two such perpendiculars, AB and CD, have been drawn. You should be able to follow this simple "proof."

Proof. $\angle ABD$ and $\angle CDB$ are both 90°. Therefore AB is parallel to CD. Therefore $\angle XAB = \angle ACD$.

The $\angle X$ is common to both the triangles AXB and CXD. The triangles are therefore equiangular and hence similar.

Therefore $$\frac{AB}{CD} = \frac{AX}{CX}$$

$$AB \times CX = AX \times CD$$

Hence $$\frac{AB}{AX} = \frac{CD}{CX}$$

So that no matter where the perpendicular is drawn, the ratio between the length of the perpendicular and one arm of the angle is always the same, and every angle will have a definite value for this ratio.

The ratio $\frac{AB}{AX}$ is the SINE of the angle.

The ratio $\frac{BX}{AX}$ is the COSINE of the angle.

The ratio $\frac{AB}{BX}$ is the TANGENT of the angle.

Their reciprocals also have special names as follows—

$\frac{AX}{AB}$ is the COSECANT of the angle.

$\frac{AX}{BX}$ is the SECANT of the angle.

$\frac{BX}{AB}$ is the COTANGENT of the angle.

If we call the angle θ (Greek "Theta") the recognized abbreviations are sin θ, cos θ, tan θ, cosec θ, sec θ, and cot θ.

Now look at the trig. tables, and take, for example, 30°. You will see that—

$$\sin 30° = 0{\cdot}5$$
$$\cos 30° = 0{\cdot}8660$$
$$\tan 30° = 0{\cdot}5774$$

The other three functions are not always given because they would make the tables unwieldy and in any case are readily found as reciprocals of sin, cos, and tan.

The functions of 0° and 90° should be noted.

$$\sin 0° = 0 \qquad \sin 90° = 1$$
$$\cos 0° = 1 \qquad \cos 90° = 0$$
$$\tan 0° = 0 \qquad \tan 90° = \infty$$

The sine and cosine can have any value between 0 and 1, but the tangent can have any value between 0 and infinity.

The relationships between the trig. functions of an angle are quite numerous and complicated equations can be formed. You need only remember these three for practical purposes.

(i) $\frac{\sin \theta}{\cos \theta} = \tan \theta$

(*Proof.*
$$\frac{\sin \theta}{\cos \theta} = \frac{AB}{AX} \div \frac{BX}{AX}$$
$$= \frac{AB}{\cancel{AX}} \times \frac{\cancel{AX}}{BX}$$
$$= \frac{AB}{BX}$$
$$= \tan \theta)$$

(ii) $\sin \theta = \cos (90° - \theta)$

$\cos \theta = \sin (90° - \theta)$

This will be clear from Fig. 17. In the triangle ABX, $\angle ABX = 90°$. Therefore $\angle AXB + \angle BAX = 90°$, since the total of the angles of any triangle is 180°. The ratio $\frac{AB}{AX}$ is therefore either sin θ or cos BAX.

Therefore $\sin\theta = \cos(90° - \theta)$.

The ratio $\frac{BX}{AX}$ is either $\cos\theta$ or $\sin BAX$.

Therefore $\cos\theta = \sin(90° - \theta)$.

(iii) $\sin^2\theta + \cos^2\theta = 1$.

(*Proof.*
$$\sin^2\theta + \cos^2\theta = \frac{AB^2}{AX^2} + \frac{BX^2}{AX^2}$$
$$= \frac{AB^2 + BX^2}{AX^2}$$

But $AB^2 + BX^2 = AX^2$, because ABX is a right-angled triangle

Hence $\sin^2\theta + \cos^2\theta = \frac{AX^2}{AX^2} = 1.$)

In radio and engineering we are very frequently interested in finding the lengths AB and BX (Fig. 16), when we know AX and the angle. This is where the trig. functions come in so usefully.

$$AB = AX\sin\theta$$
$$BX = AX\cos\theta$$

So that if AX is, say, 3 inches and θ is 30°—

$$AB = AX\sin\theta$$
$$= 3 \times 0{\cdot}5$$
$$= 1{\cdot}5 \text{ in.}$$
$$BX = AX\cos\theta$$
$$= 3 \times 0{\cdot}866$$
$$= 2{\cdot}598 \text{ in.}$$

If you now spend a few minutes considering how to find AB and BX *without* the use of trig. functions, you will see how valuable they are. The length of AX may, of course, represent some quite different quantity such as velocity or a voltage or a current.

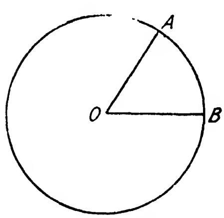

FIG. 18

Radian Measure of Angles. In addition to being measured in degrees, angles may also be measured in "radians."

A radian is an angle which has an arc of the same length as the radius.

In Fig. 18, if the length of that part of the circumference between A and B is the same as the radius OA or OB, the angle AOB is one radian. It is approximately 57°.

Since the circumference $= 2\pi \times$ radius, it follows that the whole circle of 360° is equal to 2π radians.

$$180° = \pi \text{ radians}$$

$$90° = \frac{\pi}{2} \text{ radians}$$

For scientific work in general, and radio in particular, this method of measuring angles is often more convenient than degrees.

In the trig. tables there is usually a column giving the corresponding radian measure for each value of angle in degrees. One important point should be noted—for very small angles, the radian measure, sine and tangent, have practically the same value. Thus $\frac{1}{2}°$ in radians is 0·0087 and also sin $\frac{1}{2}°$ and tan $\frac{1}{2}°$ are 0·0087.

EXERCISES IV

1. The apex angle of an isosceles triangle is 40°. What will the base angles be?

2. ABC is a right-angled triangle with the right angle at B. AB is 4·3 cms. and AC is 6·8 cms. Find BC.

3. Calculate the area of a circle with radius 2·34 inches.

4. ABC is a right-angled triangle with the right angle at B. AC is 6·2 cms. and $\angle ACB$ is 36°. Find AB and BC.

5. ABC is a triangle. AB is 2·3 inches, AC is 4·1 inches, $\angle ABC$ is 90°. Find $\angle ACB$.

6. Given that θ is $\frac{\pi}{6}$ radians, calculate the value of the expression

$$\sin \theta + 0{\cdot}32 \sin 3\theta.$$

CHAPTER V

GRAPHS

A GRAPH is a convenient pictorial method of representing the way in which one quantity varies with another, and conveys at a glance information which would take a long time to explain. A simple example of a graph is the wall-chart hung over the bed of a patient in hospital showing his or her temperature from day to day. The visiting doctor can see instantly whether any violent fluctuations have occurred or are occurring. The two quantities in this case are: (*a*) patient's temperature, and (*b*) time. Time is called the independent variable, and the patient's temperature is the dependent variable, i.e. depending on time.

A graph is usually drawn on squared paper, and has two "axes." The horizontal axis is mostly used for the independent variable and the vertical axis for the dependent variabie.

Every mathematical equation or formula can be made the subject of a graph. Taking the simplest possible equation

$$y = x$$

the graph for this would be as in Fig. 19.

The choice of scale is entirely a matter of convenience and does not affect the result. In Fig. 19 (*a*), both axes have the same scale, and in (*b*) a rather smaller scale is taken for the y axis, but, of course, both graphs are true representations of the statement $y = x$.

Straight-line graphs of this kind are called linear, and the two variables are said to have a linear relationship. The general expression for a linear graph is

$$y = ax + b$$

where a and b can have any values whatever.

In Fig. 20, $a = \frac{1}{2}$ and $b = 1$, so that the equation becomes

$$y = \tfrac{1}{2}x + 1$$

Negative Axes. If a quantity can be either positive or negative, the axes of the graph are extended beyond their point of intersection. Negative values are then measured to the left along the horizontal axis and downwards along the vertical axis.

Fig. 21 is the same as Fig. 19 (*a*), with the axes extended for negative values of x and y.

Method of Drawing Graphs. Before a graph is drawn, we have to know the following points about it.

(1) The limits of value of the independent variable.

(2) The approximate limits of the dependent variable.

(3) If the graph is of a mathematical function, the function must, of course, be stated.

As an illustration, let us construct the graph of $y = \frac{1}{x}$, between $x = +5$ and $x = -5$.

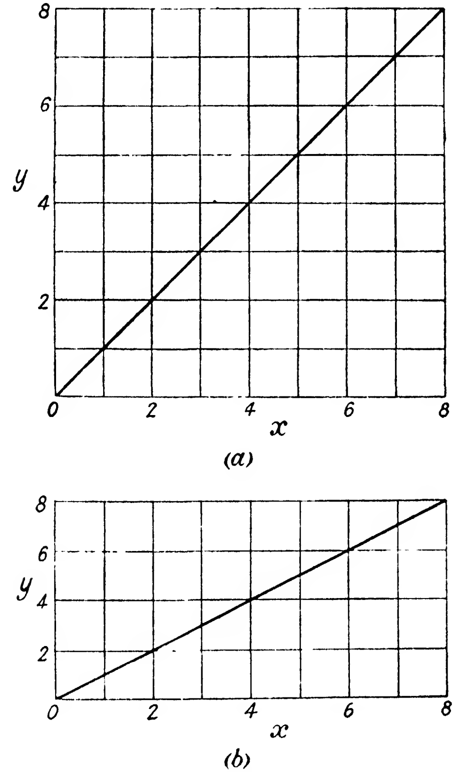

FIG. 19

First of all, put down a series of corresponding values for x and y

x	$+5$	$+4$	$+3$	$+2$	$+1$	0	-1	-2	-3	-4	-5
y	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	1	∞	-1	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{4}$	$-\frac{1}{5}$

The scale of the x-axis is easily settled because we have only to mark off five convenient units—say, inches—on each side of the zero point, which is called the origin of the graph.

The y-axis, however, presents a different problem. Looking at the figures, we see that when $x = 0$, y is infinity, meaning that the graph will go right off the paper. Further, there is an abrupt change in the value of y from 1 to infinity when x changes from 1

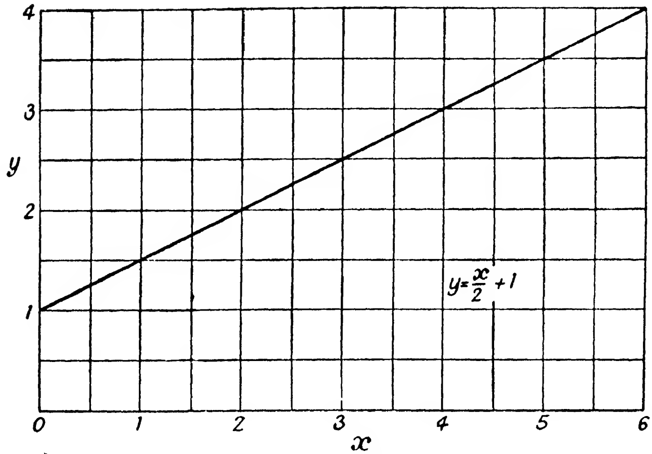

FIG. 20

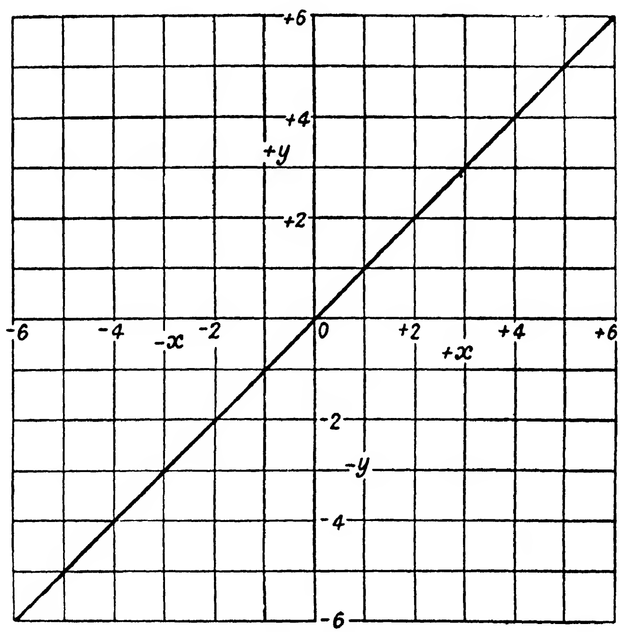

FIG. 21

to 0. We must therefore plot some intermediate points between $x = 1$ and $x = 0$.

$\pm x$	0·9	0·8	0·6	0·5	0·3	0·2	0·1
$\pm y$	1·111	1·25	1·666	2	3·333	5	10

For values of x smaller than 0·1, the value of y increases very rapidly, so a convenient limit to impose on y will be $+ 10$ to $- 10$.

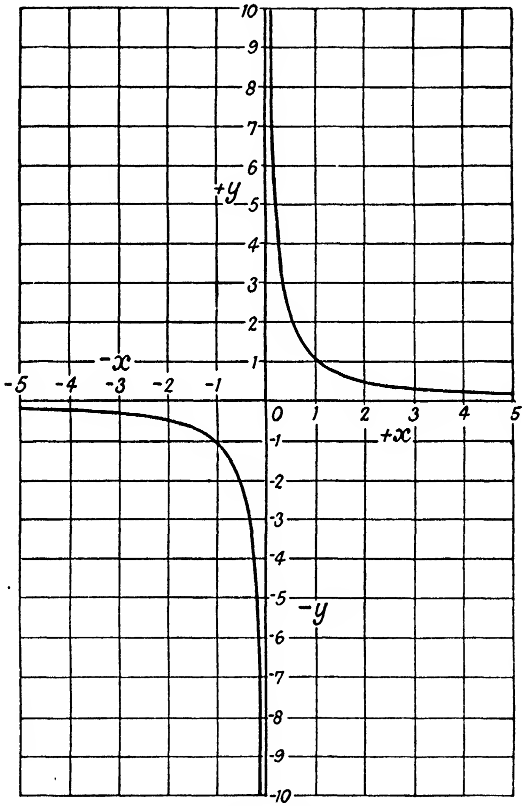

FIG. 22

Fig. 22 shows these axes with the points plotted and joined up. The graph is in two portions and goes to infinity in four directions. The curve is called a *hyperbola*.

Taking an example from electricity, the current in a circuit obeys a linear law with respect to e.m.f., but will give a hyperbola with respect to resistance.

$$I = \frac{E}{R}$$

A graph of I against E will be a straight line, and a graph of I against R will give a curve similar to that of Fig. 22.

Graph of $y = x^2$. Let us plot this from, say, $x = 4$ to $x = -4$

x	-4	-3	-2	-1	0	1	2	3	4
y	16	9	4	1	0	1	4	9	16

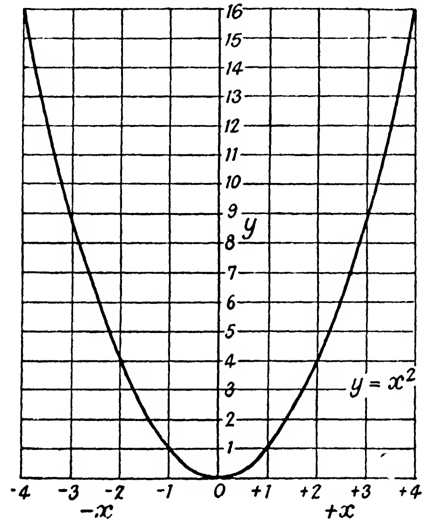

FIG 23

This graph is shown in Fig. 23, and is called a *parabola*. y is always positive because the square of a negative number is positive.

Note that when one quantity varies as the square of another, it increases very slowly at first, but then much more rapidly. This explains why hot-wire and thermo-ammeters have scales which are cramped at the zero end. The heat developed in a conductor is proportional to I^2R, so that the equation representing the action of the instrument is of the form

$$\text{Heat} = \frac{I^2Rt}{4{\cdot}2} \text{ calories}$$

i.e. it belongs to the $y = x^2$ class of graphs and will be a parabola.

Graphs of $y = \sin\ \theta$ and $y = \cos\ \theta$. These are highly important curves from a radio aspect. They are called periodic functions, because the values of $\sin \theta$ and $\cos \theta$ repeat themselves as θ increases. An angle can have any value between 0° and 360°, and there are certain conventions as to the sign (+ or —) of its sine, cosine, etc., according to the size of the angle.

In Fig. 24, a circle is shown divided into four quarters. Imagine a radius to start from the position OA and revolve anti-clockwise

round the circle. We will consider one position in each of the four quadrants. The sign convention is that distances measured to the right or upwards from the centre are positive, and distances measured to the left or downwards from the point O are negative. The following results should be clear—

$$\sin A_1OA = \frac{A_1X}{OA_1} \text{ and is positive.}$$

$$\sin A_2OA = \frac{A_2X_1}{OA_2} \text{ and is positive.}$$

$$\sin A_3OA = \frac{A_3X_1}{OA_3} \text{ and is negative.}$$

$$\sin A_4OA = \frac{A_4X}{OA_4} \text{ and is negative.}$$

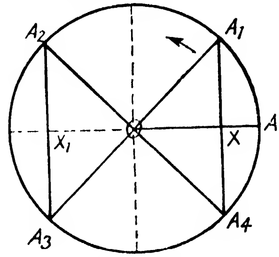

FIG. 24

That is to say, the sines of angles between 0° and 180° are positive, and the sines of angles between 180° and 360° are negative.

Examining Fig. 24 again, you will also see that—

$$\text{Cosines from} \begin{cases} 0° \text{ to } 90° \text{ are positive.} \\ 90° \text{ to } 270° \text{ are negative.} \\ 270° \text{ to } 360° \text{ are positive.} \end{cases}$$

$$\text{Tangents from} \begin{cases} 0° \text{ to } 90° \text{ are positive.} \\ 90° \text{ to } 180° \text{ are negative.} \\ 180° \text{ to } 270° \text{ are positive.} \\ 270° \text{ to } 360° \text{ are negative.} \end{cases}$$

Now we can plot $y = \sin \theta$ from the trig. tables. The θ axis will, of course, be positive only, as a negative angle has no meaning.

θ	0°	15°	30°	45°	60°	75°	90°	105°
y	0	0·2588	0·5	0·707	0·866	0·9659	1	0·9659
θ	120°	135°	150°	165°	180°			
y	0·866	0·707	0·5	0·2588	0			

θ	195°	210°	225°	240°	255°	270°
y	− 0·2588	− 0·5	− 0·707	− 0·866	− 0·9659	− 1
θ	285°	300°	315°	330°	345°	360°
y	− 0·9659	− 0·866	− 0·707	− 0·5	− 0·2588	0

Fig. 25 shows the result, which then repeats itself indefinitely if we imagine the radius OA of Fig. 24 to continue rotating. The curve for $y = \cos \theta$, which is also shown in Fig. 25, is exactly the same as the graph of $y = \sin \theta$, but is displaced from it by 90° or

a quarter period. This is because when $\theta = 0°$, $\sin \theta = 0$ and $\cos \theta = 1$—when $\theta = 90°$, $\sin \theta = 1$ and $\cos \theta = 0$, and so on.

In radio work, we deal mostly with alternating currents and voltages which vary sinoidally or very approximately so, and they are

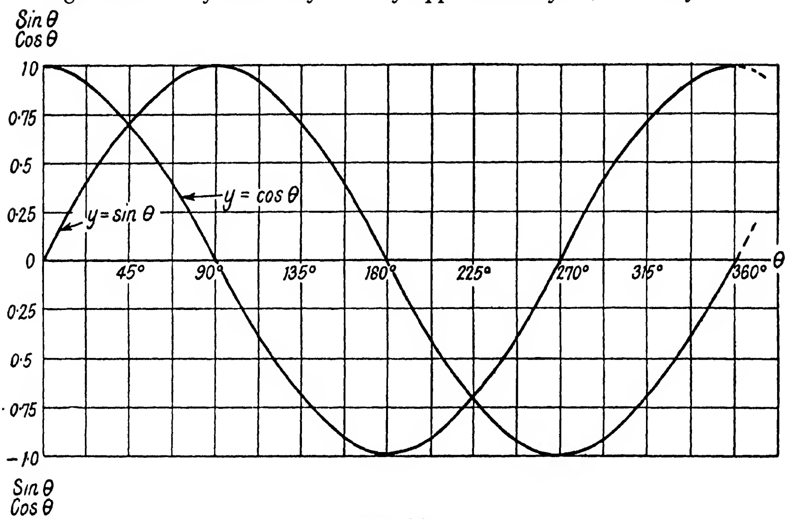

FIG. 25

represented by sine curves, as in Fig. 25, the vertical axis becoming voltage or current and the horizontal axis time.

Vector Representation. A vector is merely a straight line which represents an alternating quantity. One vector by itself has no special advantage, but when more than one alternating quantity has to be considered, it is much more convenient to represent them by vectors instead of sine curves, because the "phase" relationships can be clearly shown by drawing the vectors at the appropriate angles to each other. Thus, the vectors to replace the curves of Fig. 25 would be as in Fig. 26.

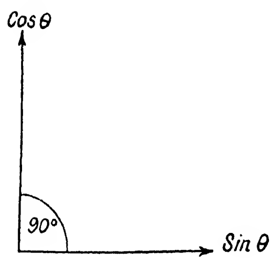

FIG. 26

A vector is conventionally assumed to rotate anti-clockwise, so that if the vertical line represents $\cos \theta$, the vector for $\sin \theta$ will be shown 90° *behind* it. This will be clear if you regard the horizontal axis of Fig. 24 as time, in which case the values of the sine will be seen to occur 90° *later* than the corresponding values of the cosine.

The length of the vector is made to show on some suitable scale either the maximum, average, or virtual values of the alternating quantity.

As an illustration of the use of vectors, consider an analysis of a simple A.C. circuit having resistance and inductance in series.

If a current I amperes is flowing in the circuit of Fig. 27, there will be an e.m.f. of IR volts across the resistance, and another of $2\pi fLI$ volts across the coil. To produce this supposed current, the supply voltage E must give a voltage equal to IR and in phase

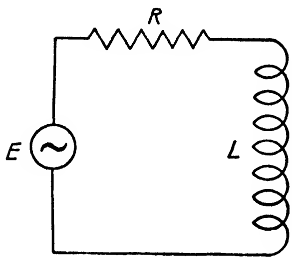

FIG. 27

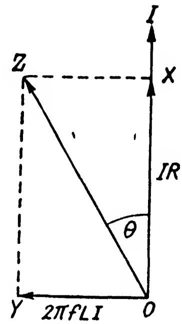

FIG. 28

with the current, and another equal to $2\pi fLI$ leading on the current by 90°.

In Fig. 28 the longer vertical vector represents the current I. OX represents IR volts and OY at 90° to OX represents $2\pi fLI$ volts. The vectorial sum of these two is found by completing the rectangular figure $OYZX$ and joining the diagonal OZ. OZ is then the vector representing the applied voltage E. Since OXZ is a right-angled triangle

$$OZ = \sqrt{OX^2 + XZ^2}$$

$$= \sqrt{I^2R^2 + (2\pi fLI)^2}$$

$$= I\sqrt{R^2 + (2\pi fL)^2}$$

or

$$I = \frac{E}{\sqrt{R^2 + (2\pi fL)^2}}$$

The phase angle θ is given by

$$\cos\theta = \frac{OX}{OZ}$$

$$= \frac{IR}{I\sqrt{R^2 + (2\pi fL)^2}}$$

$$= \frac{R}{\sqrt{R^2 + (2\pi fL)^2}}$$

or
$$\tan\theta = \frac{XZ}{OX} = \frac{2\pi fLI}{IR} = \frac{2\pi fL}{R}$$

(The method of obtaining OZ by completion of the rectangle is an example of the "Parallelogram Law" and this will be dealt with in the next chapter.)

Rate of Change. If you examine the graphs of Fig. 25 and the figures for θ and y when $y = \sin\theta$, you will see that the value of

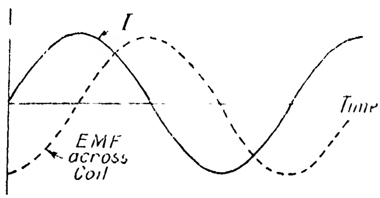

FIG. 29

$\sin\theta$ changes much more rapidly round about the values $\theta = 0°$ and $\theta = 180°$ than it does at $\theta = 90°$ and $\theta = 270°$. For instance, at each 15° on either side of 180°, $\sin\theta$ changes from 0 to $\pm$ 0·2588, but at each 15° on either side of 90° or 270°, $\sin\theta$ changes by only 0·0341. Now you will also notice that the *slope* of the graph is *greatest* at $\theta = 0°$ and $\theta = 180°$, and *least* at $\theta = 90°$ and $\theta = 270°$—in fact, at 90° and 270° the graph is at that point *parallel* to the horizontal axis and therefore has no slope. This is a very important point—that the *slope* of the graph is a direct indication of the rate at which the dependent variable is changing. Here is one example from radio. The induced e.m.f. across a coil is proportional to the rate of change of current, so that if an alternating current is flowing in a coil, the induced e.m.f. will be a maximum when the current is changing most rapidly and zero when the current is not changing.

Fig. 29 shows a sinoidal current curve. The slope of the graph is greatest when the current is passing through a zero value, and least when the current is maximum. The induced e.m.f. across the coil will therefore be maximum when the current is zero, and zero when the current is maximum, i.e. 90° out of phase with the current.

Two other illustrations of the importance of the slope of a graph are seen in static curves for valves.

Fig. 30 shows two mutual characteristics. The mutual conductance of a valve is—

$$\frac{\text{A small change in anode current}}{\text{The change in grid potential which caused it}}$$

in other words, the slope of the graph.

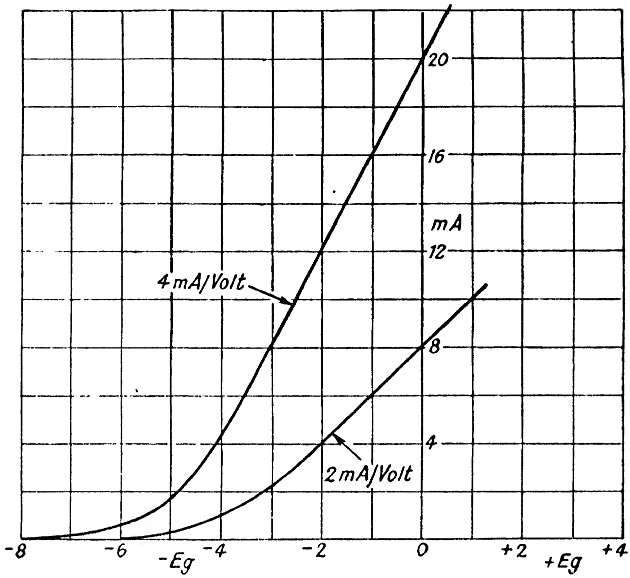
20
16
mA
12
8
4
4 mA/Volt
2 mA/Volt
-8
-6
-4
-2
0
+2
+4
-Eg
+Eg

FIG. 30

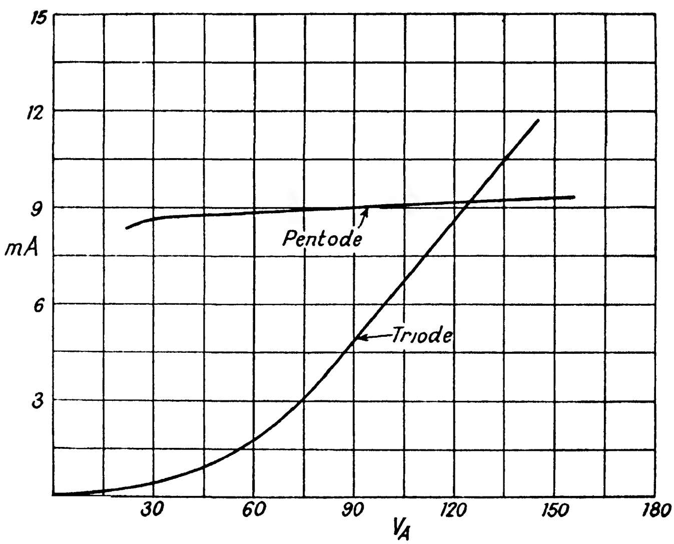
15
12
9
mA
6
3
Pentode
Triode
30
60
90
120
150
180
V_A

FIG. 31

For practical purposes, this is expressed in milliamps. per volt. In Fig. 30 one valve has a mutual conductance of 2 mA/volt and the other 4 mA/volt. The slope of the latter graph is of course much greater.

Fig. 31 shows the anode volts/anode current characteristics for a triode and pentode valve. The impedance of a valve is—

$$\frac{\text{A small change in anode volts}}{\text{The resulting change in anode current}}$$

This expression is the *reciprocal* of the slope, so that the smaller the slope, the greater will be the impedance of the valve. In Fig. 31, the triode shows an impedance of about 8000 ohms at an anode voltage of 120, and the pentode has an impedance of about 250,000 ohms at the same anode potential.

EXERCISES V

1. Sketch the graph

$$y = 2{\cdot}2x - 1{\cdot}8$$

from $x = -2$ to $x = +4$. At what point does the graph cut the x axis?

2. Using a large scale (say 5 inches for each axis) draw the graph

$$y = x^2$$

from $x = 0$ to $x = 1$.

3. Using a large scale, sketch the graph $y = \sin\theta$ from $\theta = 0°$ to $\theta = 90°$. Measure intermediate ordinates, and compare your results with the values given in a table of sines.

CHAPTER VI

MECHANICS

IF you look up the words Work, Energy, and Power in a dictionary, you will probably find that only their abstract meanings are given, i.e. the sense in which they would be used in ordinary non-technical conversation. To engineers, however, the terms Work, Energy, and Power have quite definite and concrete meanings. Work and Energy are more or less interchangeable terms, and the scientific definition of Work is as follows—

Work is done when a force moves its point of application.

Take two simple examples of this. If you hold a 1 lb. weight in your hand and raise it vertically 1 foot, you have done 1 foot-lb. of work.

If a block of wood is resting on a level table and you move it 1 foot by pushing with a force of 1 lb. you have again done 1 foot-lb. of work.

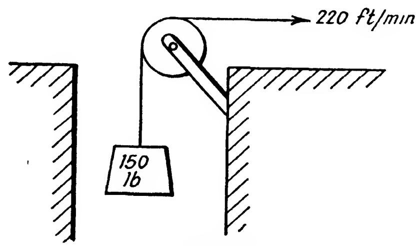

FIG. 32

In the event of the block of wood being so large that a force of 1 lb. is not sufficient to move it, then you will not have done any work, no matter for how long you apply a pressure of 1 lb.

Power. Power is the *rate* at which work is done.

The mechanical unit of power is the horse-power, which was introduced by James Watt for the purpose of having a convenient unit in which to measure the power of the newly developed steam engines.

As shown in Fig. 32, he arranged for a horse to pull a weight of 150 lb. up a vertical shaft, and found that the horse walked at the rate of 220 feet per minute. The horse therefore did work at the rate of $150 \times 220 = 33{,}000$ ft.-lb. per minute, or 550 ft.-lb. per second, and this rate of doing work is called one horse-power.

Electrical Energy and Power. The unit of electrical energy is the Joule, and can be defined as the work done when one coulomb of electricity has passed from point to point in a circuit under an e.m.f. of 1 volt.

The unit rate of doing electrical work is therefore 1 joule per second, and this is called a Watt of Power.

The relationship between electrical and mechanical power is—

$$746 \text{ watts} = 1 \text{ horse-power}$$

Conservation of Energy. It is usually the comprehension of this basic principle which distinguishes engineers from ordinary mortals. The principle of the conservation of energy can be stated thus—

Energy can neither be created nor destroyed, but can only change its form.

Let us investigate one or two cases and see what this implies. When you fire a rifle or revolver, energy is released by the explosion of the cordite. The initial form of the energy is "potential," i.e. a highly compressed gas behind the bullet. This energy is communicated to the bullet and propels it from the gun at high velocity. A large proportion of the energy is now converted to the "kinetic" energy of the flying bullet. The bullet gradually loses speed because of air friction, thus converting some of its kinetic energy to heat. When the bullet strikes some object, say a tree, its kinetic energy is expended in penetrating the resistance of the wood and is converted to heat, which, of course, is fairly quickly radiated into the surrounding atmosphere.

When you put a piece of metal in a vice and file it, both the metal and the file get hot. The energy you expend in driving the file against frictional resistance reappears as heat, and is not lost but merely radiated into the surrounding atmosphere.

When you switch on the electric light, heat and light energy appear at the bulb. This energy is conveyed along the wires from the power station as an electric current and represents a load on the dynamo. The engine turning the dynamo has to supply that energy, which in turn comes from the furnaces which are supplying heat energy for the engine.

A wireless transmitter worked by batteries converts chemical energy in the batteries to mechanical energy in the motor generator. The mechanical energy of the motor portion supplies mechanical energy to the dynamo portion, which supplies electrical energy to the h.t. circuits and hence to the aerial, where it is converted to energy in the form of an ether wave.

Every piece of apparatus, mechanical or electrical, involves some kind of energy transformation, and, in all cases, the energy which can be obtained from the machine is equal to what is put into it *minus* the inevitable "losses." Under no circumstances can we get something for nothing.

"Perpetual motion" is an impossibility in defiance of this law of nature, and even if a machine could be made to run continuously without any energy being supplied to it, it would be quite useless because the smallest load would at once stop it. To summarize the whole position, "You cannot take out energy which is not first put in."

Potential Energy. This is the energy possessed by something because of its position or because of its particular state. An example of potential energy due to position is the weight of an old-fashioned

clock. When the weight is raised, work is done on the weight, and this work is recovered by the weight driving the clock as it descends. The weight has potential energy due to its vertical height above the lowest point of its travel.

An example of potential energy due to state is a spring-driven clock or watch. When the spring is wound up it is *strained* and tends to return to its unstrained condition.

A charged condenser is an electrical example of potential energy. If C is the capacity in farads and E the voltage to which the condenser is charged, the energy stored is $\frac{1}{2}CE^2$ joules.

Kinetic Energy. This is the energy possessed by something because of its mass and velocity. Any moving object has kinetic energy. One example already mentioned is the bullet from a gun, which can penetrate the resistance of solid objects by virtue of its momentum.

If it is desired to stop a train or a motor-car, the brakes must be applied. The kinetic energy of the moving mass is then expended in overcoming the resistance of the brakes, and reappears as heat at the brake blocks and drums.

An electrical example is the energy of the magnetic field around a coil carrying a current. If L is the inductance of the coil in henries and I the current in amperes, the energy stored is $\frac{1}{2}LI^2$ joules.

Energy and Heat. From the foregoing short review of energy and power as seen by engineers, you will have gathered that energy and heat are closely connected. Heat, of course, is a form of energy, and it is in fact the form to which energy most readily changes. There must obviously be some definite relationship between heat and energy or work, but in order to state it, some arbitrary unit of heat must clearly be decided upon. There are two heat units in general use. One is called the calorie, and the other one the British Thermal Unit (B.Th.U.).

The calorie is the heat required to raise the temperature of one gramme of water by 1° Centigrade, or, alternatively, the heat given up by one gramme of water when its temperature falls by 1° Centigrade.

The B.Th.U. is the heat required to raise the temperature of 1 lb. of water by 1° Fahrenheit, or the heat given up by 1 lb. of water when its temperature falls by 1° Fahrenheit.

The actual connexion between mechanical work and heat can be determined experimentally, and the original experiments were conducted by Joule, who arranged that a known weight descending vertically through a known distance drove a paddle immersed in a known volume of water, the change in temperature of the water being observed. Much more refined methods are now possible, although Joule's figures were very close to the truth.

$$1 \text{ B.Th.U.} = 778 \text{ ft.-lb.}$$

$$1 \text{ calorie} = 3 \text{ ft.-lb. (approx.)}$$

Also, $$1 \text{ calorie} = 4{\cdot}2 \text{ joules}$$

so that $$1 \text{ joule} = 0{\cdot}72 \text{ ft.-lb. (approx.)}$$

As a final point, consider the case of a 2 kW. electric fire from the energy and heat aspect. Suppose it is worked from 200-volt electric mains. Here are all the chief facts about it, which you should study and absorb into your mental picture of the apparatus and the actions which are associated with its working.

$$\text{Wattage of fire} = 2000$$

$$\text{Mains voltage} = 200$$

$$\text{Watts} = \frac{E^2}{R}$$

$$2000 = \frac{(200)^2}{R}$$

$$R = 20 \text{ ohms}$$

$$\text{Current} = \frac{200}{20} = 10 \text{ amperes}$$

$$\text{Horse-power} = \frac{2000}{746} = 2{\cdot}68 \text{ h.p.}$$

$$= 1474 \text{ ft.-lb. per second}$$

$$2000 \text{ watts} = 2000 \text{ joules per second}$$

$$= \frac{2000}{4{\cdot}2} \text{ calories per second}$$

$$= 476 \text{ calories per second}$$

Velocity. When referring to a moving body, engineers like to use the term velocity rather than speed. It means the same thing, its only recommendation being apparently that it has four syllables instead of one.

All velocity is relative. That is to say, we cannot have any conception of velocity without reference to some object which is regarded as fixed for the purpose. For instance, if a motor-car is travelling at 30 miles an hour, we mean that it is moving at 30 miles an hour relative to the surface of the road. If it passes another car moving at 30 m.p.h. in the opposite direction, its speed relative to this car will be 60 m.p.h.

A person walking along the corridor of a train at 3 m.p.h. has this velocity relative to the train, but possibly the train has a velocity of, say, 50 m.p.h. relative to the ground. The walker's velocity relative to the ground is therefore 53 m.p.h. or 47 m.p.h. according to whether he is walking in the same direction as the train or in the opposite direction. The rails on which the train moves have a velocity relative to the centre of the earth, and the earth itself has a velocity relative to the sun. Thus an object may have any number of velocities, but of course can have only one velocity relative to one other object at any one instant.

In engineering, we are frequently concerned with what is called the Composition and Resolution of Velocities.

Composition means when we know that a body has two or more velocities relative to two or more objects, and we wish to ascertain what its velocity is relative to another object.

Resolution means when we know the velocity of a body in a certain direction and we wish to analyse this velocity into two other equivalent velocities in different directions.

Both problems are solved by means of a very important principle known as the Parallelogram Law.

The following examples will show how this is applied.

(1) *Composition of Velocities.* (*a*) A ship is travelling at 10 m.p.h. and a man walks across the deck, i.e. at right angles to the fore-and-aft line, at 4 m.p.h. Find his resultant velocity relative to the water.

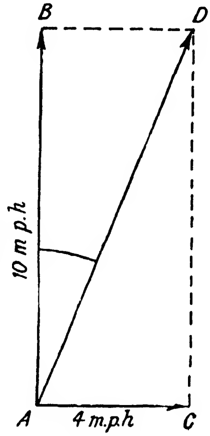

FIG. 33

In Fig. 33, AB is drawn 10 units long to represent the velocity and direction of the ship. AC, 4 units long at 90° to AB, represents the velocity and direction of the man relative to the ship. The resultant of these two is found by completing the parallelogram (in this case a rectangle) $BACD$. The diagonal AD represents the man's velocity relative to the sea both in magnitude and direction.

If the diagram is drawn accurately we can find the answer by measuring AD and the angle BAD. Alternatively, we can calculate as follows.

ABD is a right-angled triangle.

$$\begin{aligned} AD^2 &= AB^2 + BD^2 \\ &= 100 + 16 \\ AD &= \sqrt{116} \\ &= 10{\cdot}77 \text{ m.p.h.} \\ \tan BAD &= \frac{BD}{AB} = 0{\cdot}4 \end{aligned}$$

From the trig. tables, an angle whose tangent is $0{\cdot}4 = 22°$ approx.

The man's velocity relative to the sea is thus 10·77 m.p.h. at an angle of 22° to the heading of the ship.

(*b*) An aeroplane is heading true North and has a cruising speed of 200 m.p.h. There is a wind of 60 m.p.h. from 20° South of West. Find the actual track and ground speed of the aeroplane.

The aeroplane has two velocities, 200 m.p.h. true North and 60 m.p.h. in a direction 70° East of this.

Fig. 34 shows the Parallelogram Law applied to the data. AD is the resultant track, and could be calculated by "solving" the

triangle ABD. This, however, involves rather more trigonometry than can be dealt with in the present work, and it is quite sufficient to make an accurate scale drawing and measure the length of AD and the angle BAD.

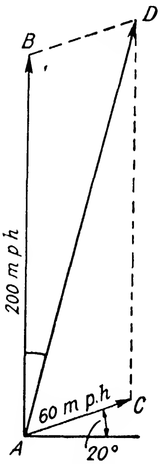

FIG. 34

It will be found that AD gives a ground speed of 226 m.p.h. and that the angle BAD is 15°. The true track of the aeroplane is thus 15° East of North at 226 m.p.h.

(2) *Resolution of Velocities.* (*a*) A person standing some distance from a straight wall starts walking towards it at 4 m.p.h. in a direction making an angle of 30° with the wall.

Find (i) his velocity towards the wall, (ii) his velocity parallel to the wall.

In Fig. 35, PQ represents the wall, and A the starting point. The line AX shows the direction in which the person walks, the angle AXP being 30°. Mark off AD on some convenient scale to represent a velocity of 4 m.p.h. Draw AM and AN at 90° to the wall and parallel to the wall respectively. From D draw DB parallel to PQ and DC parallel to AM.

$ABDC$ is now the Parallelogram of Velocities, and AB represents the person's rate of progress directly towards the wall, while AC gives his rate of progress along the wall.

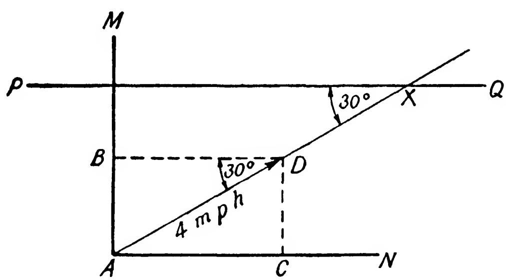

FIG. 35

Since ABD is a right-angled triangle, the solution is simple.

$$\begin{aligned} AB &= AD \sin 30° \\ &= 4 \times 0{\cdot}5 \\ &= 2 \text{ m.p.h.} \\ BD &= AD \cos 30° \\ &= 4 \times 0{\cdot}866 \\ &= 3{\cdot}464 \text{ m.p.h.} \end{aligned}$$

This problem illustrates a highly important general result, i.e. that a velocity can always be resolved into two components at 90° to each other. If V is the velocity, one component will be $V \sin \theta$, and the other one $V \cos \theta$, where θ is the angle made by one of the components with the actual direction. An electrical example will now be discussed somewhat fully.

(*b*) A conductor rotates at constant speed in a uniform magnetic field. Deduce the e.m.f. which will occur in it.

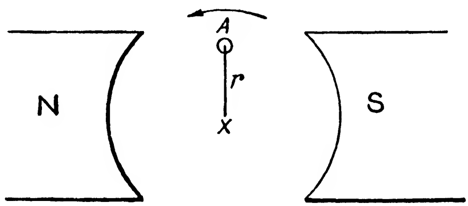

FIG. 36

Fig. 36 illustrates the conditions. A conductor A rotates at constant speed about centre X. The radius XA will be denoted by r. When an object is moving in a circular path, its direction *at any instant* is the tangent to its path at that point. Suppose the conductor makes f revolutions per second. As there are 2π radians in 360°, the conductor will describe $2\pi f$ radians every second, which is called its angular velocity. From the definition of a radian, it follows that the actual distance travelled by the conductor per second is $2\pi fr$ units—e.g. if r is expressed in feet, the linear velocity will be $2\pi fr$ ft./sec.

By Faraday's Law, the e.m.f. induced in the conductor is proportional to its velocity of cutting the magnetic flux, i.e. to that component of its velocity *at right angles* to the magnetic field. The problem thus becomes one of resolving the velocity of the conductor into two components—one at 90° to the flux and one parallel to the flux.

In Fig. 37, XA represents the starting position of Fig. 36. Now suppose the conductor is rotating and consider the state of affairs when it has moved through an angle θ to a position A_1. A_1D is drawn at 90° to the radius XA_1 and represents the direction in which the conductor is moving at that instant. The length of A_1D represents $2\pi fr$ ft./sec. on some suitable scale. Now complete the parallelogram A_1BDC, A_1B being parallel to the flux and A_1C at 90° to it. A_1B and A_1C will give the two components of velocity required. The angle BA_1D is equal to θ, proof being as follows.

A_1C and AX are parallel.

Therefore $\angle CA_1X = \theta$ (alternate $\angle$s)

$\angle DA_1X$ is 90°.

Therefore $\angle CA_1X = 90° - \angle CA_1D$

But $\angle BA_1C$ is 90°.

Therefore $\angle BA_1D = 90° - \angle CA_1D$

Thus $\angle BA_1D = \angle CA_1X = \theta$

Since A_1BDC is a rectangle $BD = A_1C$.

So that $A_1C = A_1D \sin \theta$

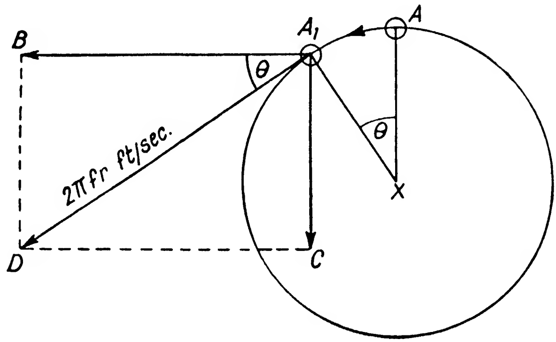

FIG. 37

As the e.m.f. is proportional to A_1C we get

E.m.f. varies as $2\pi fr \sin \theta$

$2\pi fr$ being a constant, it follows that the e.m.f. will be a sinoidal alternating voltage, and can be represented graphically as in Fig. 25.

Acceleration. Acceleration means a change of velocity. The term suggests an increasing velocity, but in the engineering sense an acceleration may be positive or negative. A negative acceleration means, of course, that the velocity is decreasing. Acceleration is measured as a rate of change of velocity. Suppose a motor-car accelerates at a steady rate from a standstill to 30 m.p.h. in 10 seconds. 30 m.p.h. is 44 ft./sec., so that the velocity has increased from 0 to 44 ft./sec. in 10 secs. The acceleration is said to be 4·4 ft. per sec. per sec. It could also be stated as 3 miles per hour per sec., but this is not customary.

A particular acceleration of great importance in engineering is that of a freely falling body. This is called gravitational acceleration and is denoted by the letter g. By a "freely" falling body is meant one not obstructed by air resistance, i.e. one falling in a perfect vacuum. Such conditions cannot, of course, be realized, but the value of g can be measured quite accurately without actually dropping anything. It is found to be 32·2 ft. per sec. per sec.

If u = initial velocity in feet per second,
a = acceleration in feet per second per second,
t = time in seconds,
v = final velocity in feet per second.

$$v = u + at$$

EXAMPLE 1. Find velocity of a freely falling body in miles per hour 4 sec. after release.

$$\begin{aligned} v &= u + at \\ &= 0 + 32{\cdot}2 \times 4 \\ &= 128{\cdot}8 \text{ ft./sec.} \\ &= \frac{128{\cdot}8 \times 60}{88} \text{ m.p.h.} \\ \text{Log. } v &= 2{\cdot}1099 + 1{\cdot}7782 - 1{\cdot}9445 \\ &= 3{\cdot}8881 - 1{\cdot}9445 \\ &= 1{\cdot}9436 \\ v &= 88 \text{ m.p.h. approx.} \end{aligned}$$

(*N.B.* It is useful to remember that 60 m.p.h. is 88 ft./sec.)

EXAMPLE 2. A train travelling at 60 m.p.h. slows down to 20 m.p.h. in 25 seconds. Find acceleration.

$$\begin{aligned} v &= u + at \\ \frac{88}{3} &= 88 - 25a \end{aligned}$$

(Expressing velocities in ft./sec. and remembering that a is negative.)

$$\begin{aligned} 25a &= 88 - \frac{88}{3} \\ a &= \frac{2 \times 88}{3 \times 25} \\ \text{Log. } a &= 2{\cdot}2455 - 1{\cdot}8751 \\ &= 0{\cdot}3704 \\ a &= -\ 2{\cdot}34 \text{ ft. per sec. per sec.} \end{aligned}$$

Force and Acceleration. When a force moves the object to which it is applied, it produces an acceleration, which may be positive or negative. For example, when a train starts from rest, the pull exerted by the engine increases the velocity from zero to some definite value. When the brakes are applied the opposing force at the brake blocks reduces the speed of the train.

The unit of Force is defined as the force which will produce unit acceleration in unit mass.

The unit of mass is the lb. and unit acceleration is 1 ft. per sec. per sec. We have seen that gravity produces an acceleration of 32·2 ft. per sec. per sec., so that to produce an acceleration of 1 ft. per sec. per sec., the force required will be $\frac{1}{32{\cdot}2} \times$ the force of gravity. This unit of force is therefore $\frac{1}{32{\cdot}2}$ lb. and is called a *Poundal*. It is, of course, just about half an ounce.

The general expression connecting Force, Mass, and Acceleration is as follows—

If P = force in poundals,

m = mass in lb.,

a = acceleration in feet per second per second.

$$P = ma$$

The poundal being a very small force, it is usually more convenient to express forces in pounds (lb.). To do this, we must divide by 32·2, so that—

$$P\text{ (lb.)} = \frac{m}{32{\cdot}2} \times a$$

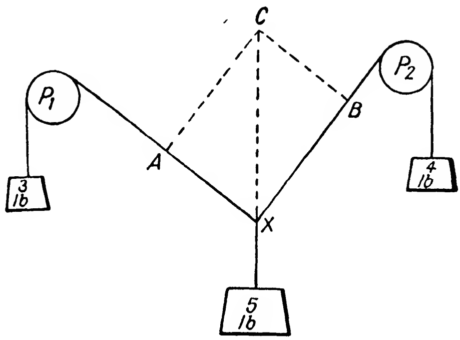

FIG. 38

This aspect of the matter gives the result that the unit of mass becomes 32·2 lb., which is often called an engineer's unit of mass. If a body weighs x lb., its mass in engineer's units is $\frac{x}{32{\cdot}2}$. No special name has been given to this unit.

EXAMPLE 3. A train weighing 300 tons accelerates at 2 ft. per sec. per sec. Find pull exerted by engine.

$$\begin{aligned} P &= ma \\ &= \frac{300 \times 2240 \times 2}{32{\cdot}2} \\ \text{Log. } P &= 2{\cdot}7782 + 3{\cdot}3502 - 1{\cdot}5079 \\ &= 6{\cdot}1284 - 1{\cdot}5079 \\ &= 4{\cdot}6205 \\ P &= 41{,}740\text{ lb.} \end{aligned}$$

Parallelogram of Forces. The Parallelogram Law can be applied to forces acting in the same plane, and Composition or Resolution of Forces is carried out in the same way as with velocities. A practical demonstration of this rule in connexion with forces can be made as shown in Fig. 38.

Weights of 3 lb., 4 lb., and 5 lb. are arranged to exert forces which act at the point X. The pulleys P_1 and P_2 are as frictionless as possible and the 5 lb. weight and the connecting threads are free to take up any position. When this has been done, it is clear that the downward pull of 5 lb. is exactly balancing the pulls exerted by the 3 lb. and 4 lb., i.e. the *resultant* of the 3 lb. and 4 lb. must be equal and opposite to the 5 lb., and the three forces are said to be in equilibrium. Measurements can be made by arranging a sheet of paper behind the strings. Mark the point X and draw lines parallel to the strings. Mark off XA equal to 3 units (say inches) and XB equal to 4 units. For the particular figures, it will be found that the angle AXB is 90°.

Complete the parallelogram $AXBC$. Since CAX is a right-angled triangle,

$$\begin{aligned} CX &= \sqrt{AX^2 + AC^2} \\ &= \sqrt{3^2 + 4^2} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

XC therefore represents the *resultant* of the forces 3 lb. and 4 lb. It will also be found that XC is vertical, i.e. in line with the direction of the 5 lb. force, which consequently balances the other two.

If different weights are chosen, the parallelogram formed may not, of course, be a rectangle, but the law will still hold good.

Simple Harmonic Motion. This kind of movement (often abbreviated to S.H.M.) is of great importance in engineering, physics, and also in radio. Even though we are not much concerned with mechanical movements in radio, an understanding of S.H.M. assists considerably in the study of an oscillatory circuit.

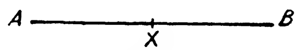

FIG. 39

Suppose that the line AB in Fig. 39 represents the path in which some object is moving backwards and forwards. The points A and B are the limits of its travel, and X is the mid-point. If the object starts from one end, say A, and then moves *with a constant acceleration towards X*, it will execute a simple harmonic motion. The exact nature of the movement is perhaps best described by means of a numerical example, in which we will trace the sequence of events.

Suppose that the constant acceleration towards X is 2 ft. per sec. per sec., and that the distance AX is such that the body takes 3 sec. to cover it.

(1) On starting from A, the body accelerates, and on reaching X it will have acquired a velocity of 6 ft./sec.

(2) At this point, the acceleration becomes negative, because the acceleration is *towards* X but the body is now moving *away* from X.

With a negative acceleration of 2 ft. per sec. per sec., the velocity of 6 ft./sec. will become zero in 3 sec., at which moment the body will have reached point B (since $BX = XA$).

(3) Direction of movement now reverses, and the body has a positive acceleration towards X, covering B to X in 3 sec., and again acquiring a velocity of 6 ft./sec.

(4) From X to A the velocity decreases at 2 ft. per sec. per sec., becoming zero at A, which restores conditions to those at starting.

This completes one swing, cycle, or vibration, whichever term may be appropriate, and with a pure S.H.M. all subsequent cycles will be repetitions.

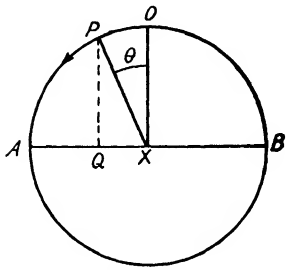

FIG. 40

A further analysis with simple mathematical treatment can be conveniently made with reference to Fig. 40.

Suppose the point P starts from O and travels at uniform speed round the circumference of the circle centre X. AB is a diameter at right angles to OX. PQ is a perpendicular to AB, i.e. angle PQA is 90°. Assume that this perpendicular follows P and consider the resulting motion of the point Q. The motion will be S.H.M., and the treatment is as follows—

In the $\triangle\ PXQ$

$$\begin{aligned} XQ &= PX \cos PXQ \\ &= PX \cos (90° - \theta) \\ &= PX \sin \theta \end{aligned}$$

PX is constant, being the radius of the circle, so that the distance XQ which is the *displacement* of the point Q from the centre X is proportional to the sine of an angle.

Calling the displacement y and PX the radius R, the equation of Q's displacement will be

$$y = R \sin \theta$$

and will be the same curve as shown in Fig. 25. We thus arrive at the highly important result that S.H.M. is a sinoidal vibration.

The velocity of the point Q in Fig. 40 will be zero at A and B and maximum as it passes through X. Reference to Fig. 37 will show that Q's velocity is represented by the component A_1B in Fig. 37, and will be $R \cos \theta$. Thus S.H.M. can be represented by a graph showing displacement against angle of rotation or velocity against angle of rotation, and in both cases, angle of rotation can be substituted by time along the horizontal axis.

In Fig. 41 (a) a free oscillating circuit is sketched, and (b) is a sinoidal graph which can represent any *one* of the following quantities on the vertical axes—

Current in the circuit.

Voltage drop across R.

Induced e.m.f. across L ($2\pi fLI$ volts).

E.m.f. across condenser $\left(\frac{I}{2\pi fC}\text{ volts}\right)$.

Charge in condenser.

The current in the circuit, i.e. the electronic movement, can be regarded as executing S.H.M., but all the other quantities also

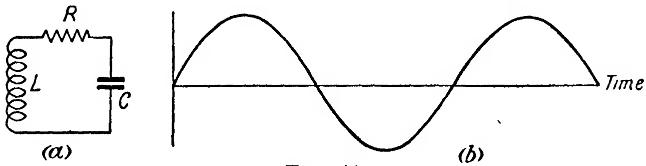

FIG. 41

go through sinoidal variations. The circuit is an electrical vibrator, and is, of course, the absolute basis of radio.

Complex Vibrations. Any kind of periodic movement (i.e. one which regularly repeats itself) can be regarded as a vibration, S.H.M. being the simplest example. A freely swinging pendulum executes S.H.M. and so do the blades of a tuning-fork when struck. The vibrations of a microphone diaphragm, a telephone ear-piece diaphragm, or the cone of a loud-speaker are usually non-sinoidal but nevertheless periodic. The pulses of current through a negatively biased valve (Class B or Class C operation) are also periodic but obviously very far from sinoidal.

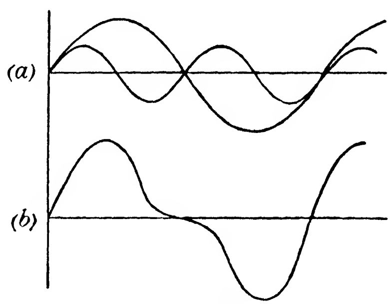

FIG. 42

One of the most important and striking facts about complex vibrations is stated in what is called Fourier's Theorem. The mathematics of a Fourier analysis are difficult and far beyond the scope of the present work, but the main implications of Fourier's Theorem are quite easy to understand.

In Fig. 42 (*a*), two sine curves are shown. One is of smaller amplitude than the other and has exactly twice its frequency. In (*b*) the two curves have been added together, i.e. by simply taking the algebraic sum of their ordinates at every point, and it will be seen that the resulting curve is also a vibration but is no longer sinoidal. If we now reverse the process, it can be seen that a vibration of the kind shown in (*b*) can be analysed into two sinoidal

vibrations, one having twice the frequency of the other. Other shaped curves may be found to consist of three, four, or more sinoidal vibrations, and Fourier's Theorem is a general statement of this fact.

Any periodic vibration can be analysed into a number of simple sinoidal vibrations, having frequencies equal to twice, three times, four times, etc., the fundamental frequency.

The above is a verbal rendering of Fourier's Theorem. As an elementary mathematical treatment, it means that any periodic vibration whatever can be represented by an equation having the form—

$$y = a \sin x + b \sin 2x + c \sin 3x + d \sin 4x + \ldots \text{ etc.}$$

Some of the terms may have zero values and are therefore missing, and the constants a, b, c, etc., may have any values, according to the nature of the vibration. As an example, take the case of Fig. 42. The higher frequency curve has a maximum value 0·35 of the lower frequency curve, so that

If $$a = 1$$
$$b = 0{\cdot}35$$

and since no other frequencies are present, all the remaining terms of the Fourier series are 0. The equation for the curve of Fig. 42 (b) therefore becomes

$$y = \sin x + 0{\cdot}35 \sin 2x$$

This is often expressed by saying that curve (b) contains 35 per cent second harmonic. A harmonic means a multiple of the fundamental frequency. Taking, for example, a frequency of 100 kcs.—

1st harmonic = 100 kcs. (fundamental)
2nd harmonic = 200 kcs.
3rd harmonic = 300 kcs.

and so on.

Any periodic vibration therefore consists of a fundamental frequency plus a number of harmonics.

Quality or "timbre" of a sound is due to the source of sound vibrating non-sinoidally and so producing harmonics or "overtones." A particular note sounded on a saxophone is readily distinguished from the same note sounded on a trumpet. The fundamental frequency is the same in each case, but a different set of harmonics is present owing to the different types of vibration in the two instruments.

The non-sinoidal pulses of anode current in a transmitting valve with negative grid bias contain harmonics, with the result that aerial current may also be non-sinoidal.

Frequency-doubling in an amplifying stage merely involves selection of the second harmonic from a non-sinoidal energy input by means of a suitably tuned circuit.

The Decibel. In radio and sound-production work, the "decibel" (abbreviation db.) has been adopted as the unit for measuring differences in signal strengths and sound intensities.

It is an arbitrary unit and has been so designed that 1 db. is about the smallest difference in sound intensity which the average person can detect. Its definition will be best seen from the following statement.

If w_1 and w_2 denote two signal strengths or sound intensities (w_1 being the greater), and n denotes their difference in decibels—

$$n = 10 \log. \frac{w_1}{w_2}$$

For a difference of 1 db. the ratio between w_1 and w_2 can be calculated as follows—

$$1 = 10 \log. \frac{w_1}{w_2}$$

$$\text{Log.} \frac{w_1}{w_2} = 0{\cdot}1$$

$$\frac{w_1}{w_2} = 1{\cdot}259$$

An increase of power of about 26 per cent thus gives an increase of 1 db.

EXAMPLE 4. An amplifier gives an overall gain of 60 db. Find the ratio of output power to input power.

$$n = 10 \log. \frac{w_1}{w_2}$$

$$\text{Log.} \frac{w_1}{w_2} = 6$$

$$\frac{w_1}{w_2} = 1{,}000{,}000$$

EXAMPLE 5. An interfering signal has a power of 15 milliwatts, and the receiver's selective devices reduce it by 20 db. What power reduction does this represent?

$$n = 10 \log. \frac{w_1}{w_2}$$

$$20 = 10 \log. \frac{15}{w_2}$$

$$\text{Log.} \frac{15}{w_2} = 2$$

$$\frac{15}{w_2} = 100$$

$$w_2 = 0{\cdot}15 \text{ milliwatts.}$$

The power reduction is therefore 14·85 milliwatts.

EXERCISES VI

1. An electric fire consists of two heating elements in parallel, each having a resistance of 40 ohms. What horse-power is taken from 210 volt mains?
2. A train is travelling at 55 m.p.h. and a stone is thrown out of a window at right angles to the train with a velocity of 30 feet per second. What is the velocity and direction of the stone relative to the railway track?
3. A person standing at the edge of a road 100 feet wide starts to walk across the road at $3\frac{1}{2}$ m.p.h. in a direction making an angle of 36° with the centre line. How long will he take to cross the road?
4. An aircraft is catapulted from the deck of a ship with an acceleration of $3g$. How long must the catapulting force last to give the aircraft a take-off velocity of 95 m.p h.?
5. A motor-car weighing 14 cwt. is accelerated by a total thrust of 450 lb. What will the acceleration be, and how long will it take for the car to reach 50 m.p.h. from a standstill, adding 6 seconds for gear-changing?
6. A water-cooled transformer is found to raise the temperature of 3400 lb. of water per hour by 100° Fahrenheit. What horse-power is thereby lost? If the transformer input is 5000 kW. what is its efficiency?

ANSWERS

EXERCISES I

1. $\frac{9}{16}$. 2. $\frac{95}{144}$. 3. $1\frac{47}{160}$. 4. $5\frac{13}{36}$. 5. $\frac{7}{48}$. 6. $\frac{11}{18}$. 7. $2\frac{2}{35}$. 8. 15·0033. 9. 0·0824. 10. 0·875, 0·65, 1·64

EXERCISES II

1. 181,700. 2. 2567 3. 0·0279. 4. 68,610. 5. 946. 6. 5·397. 7. 98·73. 8. Current = 9·78 amperes; Watts = 1936. 9. 698·1 Kcs. 10. 136. 11. 617 ohms. 12. 27·22 ohms.

EXERCISES III

1. $19a - 5b$. 2. (i) $4a^2 + 12ab + 9b^2$; (ii) $x^2 - 10xy + 25y^2$; (iii) $4p^2 - 4pq^2 + q^4$; (iv) $p^4 + 2p^2q^2 + q^4$. 3. (i) $(3a + 9b)(3a - 9b)$ or $9(a + 3b)(a - 3b)$; (ii) $(2a + b)(a - 4b)$; (iii) $(x + y)(2x + 3y)$; (iv) $(p + q)(p - q)(3p^2 + q^2)$. 4. (i) $C = \dfrac{\lambda^2}{1885^2L}$; (ii) 22·6 mmfds.

5. (i) $n = \dfrac{1 \cdot 131d \times 10^7 \times C}{AK}$; (ii) 17 plates.

6. (i) $L_1 = \dfrac{L_T L_2}{L_2 - L_T}$; (ii) 283 mics.

EXERCISES IV

1. 70° each. 2. 5·26 cms. 3. 17·2 sq. in. 4. AB = 3·64 cms.; BC = 5·01 cms. 5. 34° (approx.). 6. 0·82.

EXERCISES V

1. Graph crosses x axis at $x = 0 \cdot 818$.

EXERCISES VI

1. 2·95 horse-power. 2. 86 ft. per sec. at an angle of 20° 24′ to the railway lines. 3. 33·13 seconds. 4. 1·44 seconds. 5. 9·24 ft. per sec. per sec. 14 seconds (nearly). 6. 133·6 horse-power; 98%.

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